In More Depth: Amdahl's Law

Amdahl's law is sometimes given in another form that yields the speedup. *Speedup* is the measure of how a computer performs after some enhancement relative to how it performed previously. Thus, if some feature yields a speedup ratio of 2, performance with the enhancement is twice what it was before the enhancement. Hence, we can write

$$\text{Speedup} = \frac{\text{Performance after improvement}}{\text{Performance before i m provement}} = \frac{\text{Execution time before improvement}}{\text{Execution time after im provement}}$$

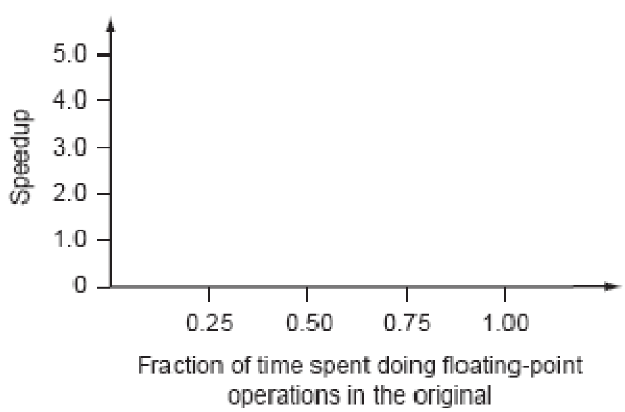
The earlier version of Amdahl's law was given as

$$\text{Execution time after improvement} = \frac{\text{Execution time affected by improvement}}{\text{Amount of improvement}} + \text{Execution time unaffected}$$

Exercise 1. Consider some benchmark computation consisting of floating-point (fp) operations and non-fp operations. Let's look at how speedup behaves when we incorporate the faster floating-point hardware of a computer. Suppose the total execution time of the benchmark task before the floating-point enhancement is 10 seconds, of which 60% is spent on executing floating-point instructions. What will the speedup be if we enhance the computer to make all floating-point (fp) instructions run 5 times faster?

Exercise 2. We are testing some benchmark tasks to show off performance of floating-point units (comparing old and new floating-point hardware). One benchmark we are considering runs for 100 seconds (total initial execution time) with the old floating-point hardware. We observed the overall benchmark showing a speedup of 3 due to the floating-point enhancement with a new floating-point unit running 5 times faster. How much of the initial execution time would floating-point instructions have to account for?

Exercise 3. Assume that we enhance the floating-point unit which is running 5 times faster. Plot the speedup obtained, versus the fraction of original execution time in some benchmark task spent doing floating-point operations, on a graph of the following form:



Exercise 4. You are going to enhance a computer, and there are two possible improvements: either make multiply instructions run 4 times faster than before, or make memory access instructions run 2 times faster than before. You repeatedly run a program that takes 100 seconds to execute. Of this time, 20% is used for multiplication, 50% for memory access instructions, and 30% for other tasks. What will the speedup be if you improve only multiplication? What will the speedup be if you improve only memory access? What will the speedup be if both improvements are made?

Exercise 5. You are going to change the program described in Exercise 4 so that the percentages are not 20%, 50%, and 30% anymore. Assuming that none of the new percentages is 0, what sort of program would result in a tie (with regard to speedup) between the two individual improvements? Provide both a formula and some examples.

Exercise 6. Amdahl's law is often written in terms of overall speedup as a function of two variables: the size of the enhancement (or amount of improvement) and the fraction of the original execution time that the enhanced feature is being used. Derive this form of the equation from the two equations above.