Lecture 5: Arithmetic and Logical instructions



There are 10 types of people in the world: those who understand binary, and those who don't.

Integer numbers

Topics

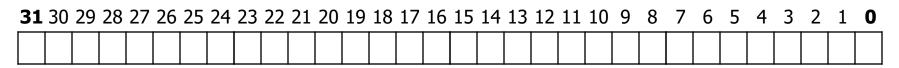
- MIPS arithmetic and logical instructions
- Bits masking example (lab 7)
- Some textbook references
 - PH Ed3: 3.1, 3.2, 3.3, 3.5
 - PH Ed4: 3.1, 3.2, 3.3, 3.5
 - PH Ed5: 3.1, 3.2, 3.3, 3.5
 - PH Ed6: 3.1, 3.2, 3.3, 3.5

3-bit		Decimal Value	2S	
Binary	Sign	1's Complement	2's Complement	
pattern	Magnitude	•if MSB=0, positive value	•if MSB=0, positive value	
		•if MSB=1, invert bits, assume negative	•if MSB=1, invert bits, add 1, assume negative	
000	+0	+0	+0	
001	+1	+1	+1	
010	+2	+2	+2	
011	+3	+3	+3	
100	-0	-3	-4	
101	-1	-2	-3	
110	-2	-1	-2	
111	-3	-0	-1	

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Interpreting bit patterns

- A 32-bit word has no inherent meaning; it can represent various things:
 - ?
 - ?
 - ?
- Bits in a word always are numbered from right to left
 - Least Significant Bit (LSB) bit 0 (rightmost)
 - Most Significant Bit (MSB) bit 31 (leftmost)



Unsigned binary number

- Representation
 - straightforward for natural numbers
- Example
 - 10110 has a decimal value
 - $(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 22$

2 ⁴	2 ³	2 ²	2 ¹	2 ⁰
1	0	1	1	0

- Given an n-bit number
 - Range: 0 to 2ⁿ 1 (2ⁿ different numbers)
 - Using 3 bits: 0 to 7 000 001 010 011 100 101 110 111 2 3 4 5 7 6 • $\mathbf{x} = \mathbf{x}_{n-1}\mathbf{2}^{n-1} + \mathbf{x}_{n-2}\mathbf{2}^{n-2} + \dots + \mathbf{x}_1\mathbf{2}^1 + \mathbf{x}_0\mathbf{2}^0$

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Signed binary number

5 6 2 3 4 7 8 9 10 11 12 13 14 15 0000 0010 0100 0110 1000 1010 1100 1110 0001 0011 0101 0111 1001 1011 1101 1111

- We need both positive numbers and negative numbers
- How do we distinguish between them?
 - Turn some UNSIGNED numbers into negative numbers
 - Options? [e.g. +8 as 0? +8 as -1? +15 as 0? +15 as -1? ...]
- The obvious solution would be:
 - Reserve one bit for sign, then sign and magnitude representation
 - Symmetry around zero
 - same number of positive and negative numbers represented [0000/1000; 0001/1001; 0010/1010; ...]
 - but we have two zeros [1000₂ as -0 in the example above]

Bias representation - 1's complement

4 5 6 7 2 3 9 10 11 12 13 14 15

Give up on symmetry

- [0000/1000; 0001/1001; 0010/1010; ...]

Translation of negative range by adding a distance (bias)

 $representation(x) = \begin{cases} Binary (x) & \text{if } 0 \le x < 2^{n-1} \\ Binary (bias - |x|) & \text{if } -2^{n-1} < x < 0 \end{cases}$

- 1's complement
 - if we select **bias** = **2ⁿ-1**, we get 1's complement representation

Bias representation - 1's complement

- -6 -5 -4 -3 -2 -1 -0 10 11 12 13 14 15
- Note
 - no value is mapped to ±2ⁿ⁻¹; there are two 0s
 - pattern of all 1's is commonly referred to as negative zero
 - but we have symmetry
- Decimal Value of a negative number (e.g. 1010)
 - MSB determines the sign
 - Invert all bits, get the value for the positive number 1010 -> inverted 0101 -> 5
- Problems
 - Arithmetic operations: try (-3) + (-4)
 1011 +

Bias representation - 2's complement

- Translation of negative numbers by a distance (**bias**) representation(x) = $\begin{cases} Binary (x) & \text{if } 0 \le x < 2^{n-1} \\ Binary (bias - |x|) & \text{if } -2^{n-1} \le x < 0 \end{cases}$
- 2's complement
 - if we select bias = 2ⁿ, we get 2's complement representation
- Note
 - we can represent a range from -2ⁿ⁻¹ to 2ⁿ⁻¹ 1
 - results in the simplest (fastest) hardware
 - universally accepted in all modern computers (also MIPS)

Bias representation - 2's complement

- Decimal Value of a negative number (e.g. 1010)
 - MSB determines the sign
 - Invert all bits, and add one , get the value for the positive number
 1010 -> inverted 0101 (5) -> 5 + 1 = 6
- Advantage
 - Arithmetic operations work naturally: try (-3) + (-4)

 $\frac{1101}{1100} + \frac{11001}{11001}$

- Sign extension
 - When moving n bits into an n+m bits container, it's safe to extend the sign bit to the leftmost





00000	0 0 0	0 0 0	0 0	0 0 0	000	0 0	0 0	0 0	0 0	0 0 1	01
1 1 1 1 1	1 1 1	1 1 1	1 1	1 1 1	1 1 1	1 1	. 1 1	1 1	1 1	1 1 1	0 1

Bias representation - 2's complement

- 2's complement negation
 - Given x -> obtain -x
 - invert the number (turn every 0 to 1, and 1 to 0) ~x
 - Then add 1, that is -x = -x + 1
- Two's complement operations: Addition & Subtraction
 - addition the same as for unsigned numbers
 0101
 5
 <u>+1010
 1111
 -1
 </u>
 -1
 - subtraction using addition of negative numbers

0101	0101	5
- 1010	+ 0110	<u>+ 6</u>
	1111	11

Overflow [Read more from the textbook]

- Overflow (result too large for finite computer word):
 - e.g., adding two n-bit numbers does not yield an n-bit number
 - the computer word is finite
- Two choices:
 - ignore overflow: for example in address arithmetic
 - detect and handle overflow in hardware
 - set a flag (overflow register)
 - or exception in the execution of the program

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Handling overflow

- Detecting Overflow
 - No overflow is possible when
 - Addition: a positive and a negative number
 - Subtraction: signs are the same
 - Overflow occurs when the value affects the sign
 - adding two positives yields a negative
 - adding two negatives gives a positive
 - subtract a negative from a positive and get a negative
 - subtract a positive from a negative and get a positive
- Handling overflow
 - Overflow register
 - not in modern RISC architectures (MIPS there is no such a register)
 - An exception is triggered by hardware
 - in MIPS a special purpose register EPC (Exception Program Counter) can be used (details later)

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Ignoring overflow

- We don't always want to detect overflow
 - When running unsigned operations
 - MIPS instructions: addu, addiu, subu, sltu, ...
- Note:
 - With addu, the "u" means "don't trap overflow"
 - addiu and sltiu still sign-extend
 - sltu for unsigned comparisons

Summary of Representations

3-bit		Decimal Values						
Binary	Sign	1's Complement	2's Complement					
pattern	Magnitude	•if MSB=0, positive value	•if MSB=0, positive value					
		 if MSB=1, invert bits, assume negative 	•if MSB=1, invert bits, add 1, assume negative					
000	+0	+0	+0					
001	+1	+1	+1					
010	+2	+2	+2					
011	+3	+3	+3					
100	-0	-3	-4					
101	-1	-2	-3					
110	-2	-1	-2					
111	-3	-0	-1					

Unsigned and signed instructions

- A number can be interpreted by hardware as signed or unsigned
 - A byte may be an ASCII character, or of some other meaning
 - it depends only on the instruction operating on the number
- MIPS provides instructions for signed and unsigned numbers

	Signed	Unsigned
arithmetic	add, addi, sub, mult, div	addu, addiu, subu, multu, divu
comparison	slt, slti	sltu, sltiu
load	lb, lh	lbu, lhu

- Answer these questions:
 - why don't we have two versions of the lw instruction?
 - why don't we have two versions of the store byte sb instruction?

Unsigned and signed instructions

- example:
 - \$s0: 1111 1111 1111 1111 1111 1111 1111
- Answer these questions:
 - what is the value of \$t0 and \$t1?
 slt \$t0,\$s0,\$s1 #
 sltu \$t1,\$s0,\$s1 #

MULTIPLY in MIPS: Instructions

- MIPS registers
 - two special purpose registers hi and lo
 - hi: high-order word of product
 - Io: low-order word of product
- MIPS instructions

mult rs1, rs2 # (hi, lo) = rs1 * rs2 ;signed multu rs1, rs2 # (hi, lo) = rs1 * rs2 ;unsigned mfhi rd # move from hi to rd mflo rd # move from lo to rd

Pseudo instructions

mul \$t0,\$s1,\$s2
mulo \$t0,\$s1,\$s2

DIVIDE in MIPS: Instructions

 all divide instructions put Remainder into hi register, and Quotient into lo register

- Overflow and division by 0 are NOT detected by hardware
 - software takes responsibility
 - assembly language programmer or compiler
- Pseudo instructions

div \$t0,\$s1,\$s2

Logical operations

we may want to interpret a word

- as fields of bits of various lengths
- including a series of single bits



- instructions for operating on bit fields
 - shifts logical operations
 - bitwise logical operations

Shifts (Logical shifts, Arithmetic shift)

Logical shifts

- move all the bits in the register to the left or to the right filling the empty space with zeros
- bits "shifted-out" are lost
- shamt (shift amount): constant
- Put the result in register rd:

```
sll rd,rt,shamt # shamt is a constant
sllv rd,rt,rs # Shift left logical variable
srl rd,rt,shamt #
srlv rd,rt,rs #
```

Shifts (Logical shifts, Arithmetic shift)

Arithmetic shift

shift to the right with sign extension

```
sra rd,rt,shamt # shamt is a constant
#
srav rd,rt,rs # sra by a variable number of bits
```

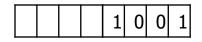
- Answer this question:
 - why no arithmetic shift to the left?

Rotation

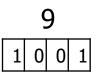
- ror, rol
- pseudoinstructions to rotate the register to left or right by a number of bits
- no bits lost, bits "falling off" one end fed into the other end

Arithmetic by shifting

- For a base *n* representation
 - a shift to the left is like multiplying by n



- sll rd, rs, 2
- a shift to the right is like dividing by n
- PITFALLS
 - multiplying numbers by shifting left may result in overflow
 - but can be used with caution for small integers, for example
 - division by arithmetic (not logical) right shift
 - positives rounded down



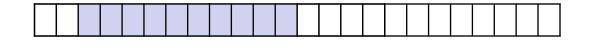
negatives? also rounded down?

Logical bitwise operations

- performed bit by bit, so called bitwise operations
- general format like addition
 - log-op rd, rs1, rs2
 # R-type instruction
 - log-opi rd, rs, constant # I-type instruction

0 0 1 1 1 1

- instructions available in MIPS (examples will follow)
 - logical AND
 - logical OR
 - logical NOR
 - logical XOR



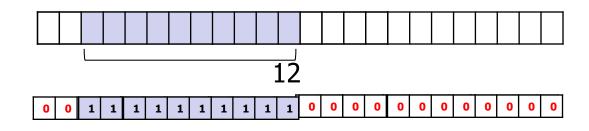
0 0 0 0 0 0 0 0 0

1 1 1 1 1 1 1 1

exercise: think about this: ...why don't we have unsigned logical instructions? 0



- "cutting" out bit fields from a word
- a mask is a word (a constant or register contents)
 - with "1" for bits we want to keep
 - with "0" for bits we want to discard
- a logical AND on the mask and a word
 - leaves only the bits we selected in the mask
 - all other bits are cleared (replaced with zeros)



Extracting fields in an instruction

ASSUME: register \$s1 contains a R-type instruction TASK: extract the register numbers rs, rt, rd used in the instruction and save them in registers \$s2, \$s3, \$s4 respectively

\$s1	ор	rs	rt	rd	shamt	funct
	6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

Two approaches

- shift first, mask second
- mask first, shift second
- We will mask first
- all masks happen to be 5-bit long, so we can shift masks

Extracting fields in an instruction

addi \$t0,\$zero, 0z	(1800 # mas)	k for rd
and \$s4,\$s1,\$t0	# ext:	ract the field
srl \$s4,\$s4, 11	# righ	nt alignment
sll \$t0,\$t0, 5	# mas]	k for rt
and \$s3,\$s1,\$t0	#	
srl \$s3,\$s3,16	#	
sll \$t0,\$t0,5	# mas]	k for rs
and \$s2,\$s1,\$t0	#	
srl \$s2,\$s2,21	#	

\$s1	ор	rs	rt	rd	shamt	funct
	6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

Extracting 2's complement numbers

ASSUME: \$s1 contains THREE 10-bit long 2's complement numbers, packed in bits 2 to 31

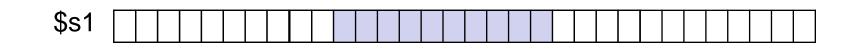
TASK: let's extract the middle number



We know:

- the number is 10-bit long
- the number starts at bit position 12
- Strategy
 - 10-bit mask (for bits 0-9) is 0x0000 03ff
 - Left-shift 0x0000 03ff by 12 to generate the mask needed
 - Sign extension is needed

Extracting 2's complement numbers



addi \$t0,\$zero, 0x03ff # 10-bit mask (for bits 0-9)

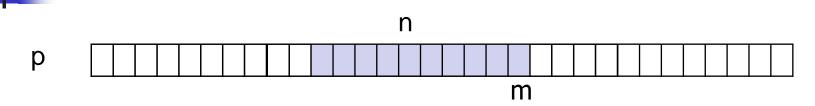
sll \$t0,\$t0, 12

and \$s2,\$s1,\$t0

sll \$s2,\$s2, 10 sra \$s2,\$s2, 22

- # Left-shift by 12 to generate
 # the mask needed
 #
- # left most to touch MSB
- # sign extension

Enlarged bit patterns from the previous page

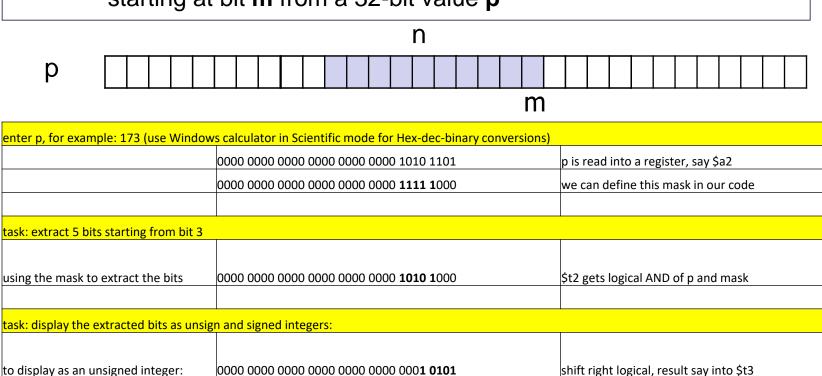


0000 0000 0000 0000 0000 0000 1010 1101	p=173
0000 0000 0000 0000 0000 0000 1111 1 000	mask
0000 0000 0000 0000 0000 0000 1010 1 000	AND
0000 0000 0000 0000 0000 0000 000 1 0101	srl
1010 1 000 0000 0000 0000 0000 0000 000	sll
1111 1111 1111 1111 1111 1111 1111 111 1 0101	sra

LAB 7 help: write procedure "extract"

Refer to maskinghelp.pdf in 'Extra Materials' ribbon on vUWS

ASSUME: Numbers entered from keyboard are p, m, n TASK: Write procedure named "extract" which extracts an **n**-bit field starting at bit **m** from a 32-bit value **p**



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0000 0000 0000 0000 0000 0000 000**1 0101**

1111 1111 1111 1111 1111 1111 1111 111**1 0101**

to display as a signed integer do two shifts:

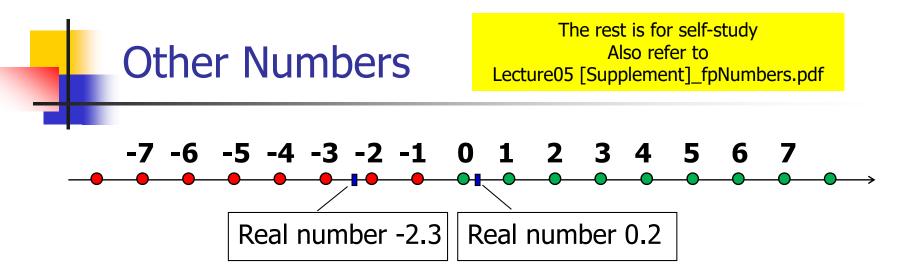
first shift

second shift -- final result

shift right logical, result say into \$t3

shift right arithmetic back to bit 0

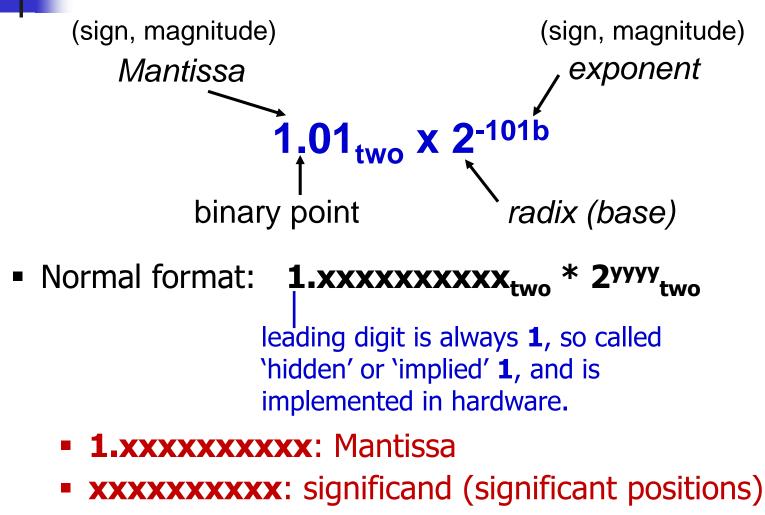
shift left logical up to bit 31



- What about
 - Very large numbers? (seconds/century) 3,155,760,000_{ten} (3.15576_{ten} x 10⁹)
 - Very small numbers? (second / nanosecond) 0.00000001_{ten} (1.0_{ten} x 10⁻⁹)
 - Rationals
 - 2/3 (0.666666666...)
 - Irrationals
 - 2^{1/2} (1.414213562373...)
 - Transcendentals

e (2.718...), π (3.141...)

Scientific Notation for Binary Numbers



• **yyyy**: exponent

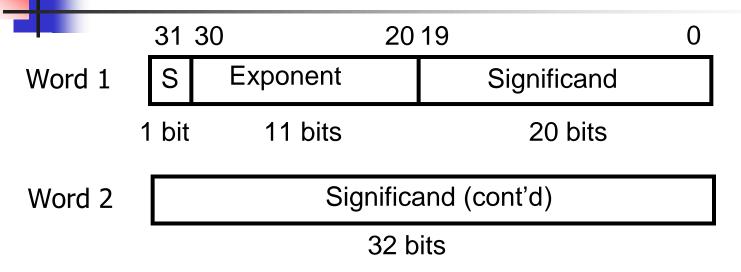
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IEEE 754 Floating Point Standard

31 30		23 22		0
S	Exponent		Significand	
1 bit	8 bits		23 bits	

- Word Size (32 bits, 23-bit Significand Single Precision)
- Value: (-1)^s x Mantissa x 2^{Exponent} [broken into 3 parts]
- Range: Represent numbers as small as 2.0 x 10⁻³⁸ to as large as 2.0 x 10³⁸
 - if result too large? (> 2.0x10³⁸), Overflow => Exponent larger than can be represented in 8-bit Exponent field
 - if result too small? (>0, < 2.0x10⁻³⁸), Underflow => Negative exponent larger than can be represented in 8-bit Exponent field
- Issues: increase range (Exponent field) and accuracy (no. of significant positions)

IEEE 754 Floating Point Standard

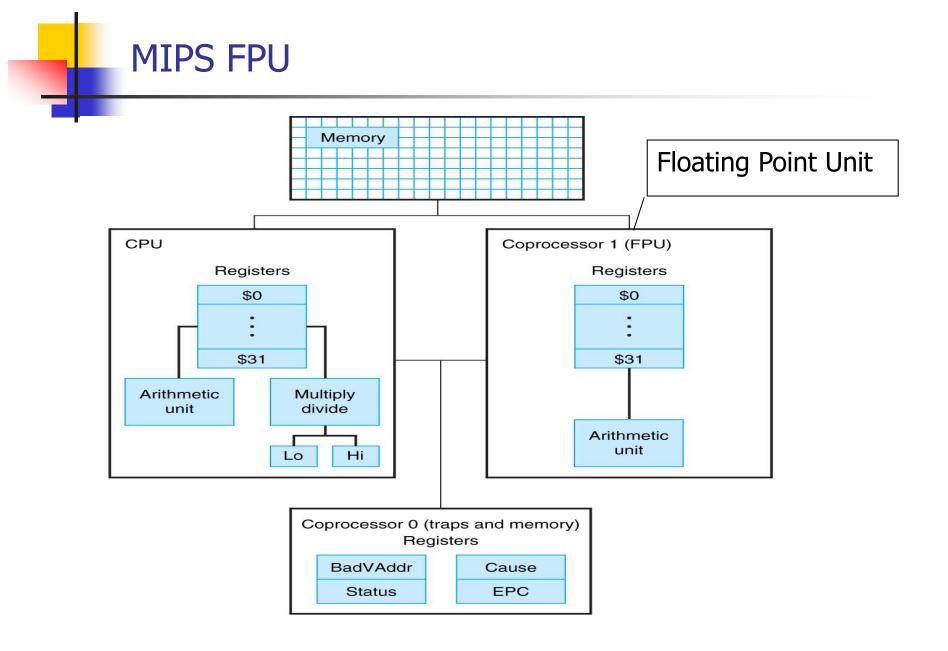


- Multiple of Word Size (64 bits, 52-bit Significand for Double Precision)
- Representing Mantissa: If significand bits left-to-right are s₁, s₂, s₃, ... then, Mantissa: 1.s₁s₂s₃...; the FP value is:

 $(-1)^{S} \times (1 + (s_1 \times 2^{-1}) + (s_2 \times 2^{-2}) + (s_3 \times 2^{-3}) + ...) \times 2^{Exponent}$

NOTE: $1.s_1s_2s_3...$

20	2 -1	2 -2	2 -3	2-4	2 -5	
1	S ₁	S ₂	S ₃	S 4	S 5	



MIPS Floating Point Architecture

- Single Precision, Double Precision versions of add, subtract, multiply, divide, compare
 - Single add.s, sub.s, mul.s, div.s, c.lt.s
 - Double add.d, sub.d, mul.d, div.d, c.lt.d
- Registers
 - Simplest solution: use existing registers
 - Normally integer and FP operations on different data, for performance could have separate registers
- MIPS provides 32 32-bit FP. reg: \$f0, \$f1, \$f2 ...,
 - Thus need FP data transfers: lwc1, swc1
 - Double Precision? Even-odd pair of registers (\$f0#\$f1) act as 64-bit register: \$f0, \$f2, \$f4, ...

New MIPS FP arithmetic instructions

add.s \$f0,\$f1,\$f2 # \$f0=\$f1+\$f2 FP Add (single) add.d \$f0,\$f2,\$f4 # \$f0=\$f2+\$f4 FP Add (double) sub.s \$f0,\$f1,\$f2 # \$f0=\$f1-\$f2 FP Subtract (single) sub.d \$f0,\$f2,\$f4 # \$f0=\$f2-\$f4 FP Subtract (double) mul.s \$f0,\$f1,\$f2 # \$f0=\$f1x\$f2 FP Multiply (single) mul.d \$f0,\$f2,\$f4 # \$f0=\$f2x\$f4 FP Multiply (double) div.s \$f0,\$f1,\$f2 # \$f0=\$f1÷\$f2 FP Divide (single) div.s \$f0,\$f1,\$f2 # \$f0=\$f1÷\$f2 FP Divide (single) div.d \$f0,\$f2,\$f4 # \$f0=\$f2÷\$f4 FP Divide (double) c.X.s \$f0,\$f1 # flag1= \$f0 X \$f1 FP Compare (single) # flag1= \$f0 X \$f2 FP Compare (double)

```
# where X is: eq (equal), lt (less than), le (less than
# equal) to tests flag value:
# bclt - floating-point branch true
# bclf - floating-point branch false
```

Example with FP Multiply [Exercise - homework]

- }
- Starting *addresses* are parameters in \$a0, \$a1, and \$a2.
 Integer *variables* are in \$t3, \$t4, \$t5. Arrays 32 by 32
- Use pseudoinstructions: li (load immediate), l.d/s.d (load/store 64 bits)

MIPS code for first piece: initilialize, x[][]

Initailize Loop Variables

mm:	• • •			
	li \$t1,	32 #	\$t1 = 32	
	li \$t3,	0 #	i = 0; 1st loc	op
L1:	li \$t4,	0 #	j = 0; reset 2	2 nd
L2:	li \$t5,	0 #	k = 0; reset 3	3rd

To fetch x[i][j], skip i rows (i*32), add j

sll \$t2,\$t3,5 # \$t2 = i * 2⁵
addu \$t2,\$t2,\$t4 # \$t2 = i*2⁵ + j

Get byte address (8 bytes), load x[i][j]

sll \$t2,\$t2,3	<pre># i,j byte addr.</pre>
addu \$t2,\$a0,\$t2	# @ x[i][j]
l.d \$f4,0(\$t2)	# \$f4 = x[i][j]

MIPS code for second piece: **z[][], y[][]**

- Like before, but load z[k][j] into \$f16
 - L3: sll \$t0,\$t5,5 # \$t0 = k * 25 addu \$t0,\$t0,\$t4 # \$t0 = k*25 + j sll \$t0,\$t0,3 # k,j byte addr. addu \$t0,\$a2,\$t0 # @ z[k][j] l.d \$f16,0(\$t0) # \$f16 = z[k][j]
- Like before, but load y[i][k] into \$f18

sll \$t0,\$t3,5	#	\$t0 = i * 25
addu \$t0,\$t0,\$t5	#	t0 = i*25 + k
sll \$t0,\$t0,3	#	i,k byte addr.
addu \$t0,\$a1,\$t0	#	0 y[i][k]
l.d \$f18,0(\$t0)	#	\$f18 = y[i][k]

Summary: \$f4:x[i][j], \$f16:z[k][j], \$f18:y[i][k]

MIPS code for last piece: add/mul, loops

Add y*z to x

mul.d \$f16,\$f18,\$f16 # y[][]*z[][]
add.d \$f4, \$f4, \$f16 # x[][]+ y*z

Increment k; if end of inner loop, store x

addiu \$t5,\$t5,1	# k = k + 1
bne \$t5,\$t1,L3	# if(k!=32) goto L3
s.d \$f4,0(\$t2)	# x[i][j] = \$f4

Increment j; middle loop if not end of j

addiu \$t4,\$t4,1	# j = j + 1
bne \$t4,\$t1,L2	# if(j!=32) goto L2

Increment i; if end of outer loop, return

addiu \$t3,\$t3,1	# i = i + 1
bne \$t3,\$t1,L2	# if(i!=32) goto L1
jr \$ra	

Revision quiz

A binary pattern 1010 in 2's complement has equivalent decimal value:

1) -6 **2)** 10 **3)** 16

- Is the following statement correct?
 A 32-bit word, without specifying a context, has no inherent meaning. That is, it can represent various things.
- sll \$s2, \$s1, 1 has the same effect as
 1) add \$s2, \$s1, \$s1
 2) sub \$s2, \$s1, \$s1
 3) muli \$s2, \$s1, 1

Recommended readings

General Data Extra Materials

<u>UnitOutline</u> | <u>LearningGuide</u> | <u>Teaching Schedule</u> | <u>Aligning Assessments</u> | <u>ascii_chart.pdf</u> | <u>bias_representation.pdf</u> | <u>HP_AppA.pdf</u> | <u>instruction decoding.pdf</u> | <u>masking help.pdf</u> | <u>PCSpim.pdf</u> | <u>PCSpim Portable Version</u> | <u>Library materials</u>

PH6, §3.1, §3.2, §3.3, §3.5: MIPS Arithmetic; MIPS FP Architecture PH5, §3.1, §3.2, §3.3, §3.5 [p211-p217 of §3.5]: MIPS Arithmetic; MIPS FP Architecture PH4, §3.1, §3.2, §3.3, §3.5 [p259-p265 of §3.5]: MIPS Arithmetic; MIPS FP Architecture

HP_AppA.pdf -> A-51: Arithmetic and Logical Instructions

Text readings are listed in Teaching Schedule and Learning Guide

PH6 (PH5 & PH4 also suitable): check whether eBook available on library site

PH6: companion materials (e.g. online sections for further readings)

https://www.elsevier.com/books-andjournals/bookcompanion/9780128201091

PH5: companion materials (e.g. online sections for further readings) http://booksite.elsevier.com/978012407 7263/?ISBN=9780124077263