Online Market Coordination

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Abstract. The aim of this paper is to deal with the problem of coordination for online markets where a seller registers to an online market to sell an item. The seller and the owner of the market then form an alliance to generate revenue through online sales. However, the efficiency and stability of the alliance highly relies on the contract that specifies the way to split the revenue and costs over the alliane members. We consider some typical contracts and examine their influences on the behavior of the alliance. We introduce the key concept of alliance coordinattion which characterizes the efficiency and stability of an online market.

1 Introduction

The explosive growth of online markets (or e-markets) has caused many changes in the way business is done traditionally. An online market is a web-based facility with which multiple traders can sell or buy goods and services through the Internet. Typical online markets are eBay, Amazon, lastminute.com, and so on. Different from the traditional markets, the partnership between traders and the market owner in an online market can be loosely tied and dynamic in most situations. A trader can enter the market any time and could leave the market any time also even during a transaction. The market owner has poor information about the traders. In addition, an online market can normally accommodate thousands of traders to trade in the market. Monitoring the behavior of each trader is hard. Therefore the mechanism that coordinates the market owner and traders in an online market is critical to the efficiency, effectiveness and stability of the market.

We consider a typical situation whereby a seller registers to an online market to sell a certain product. The seller and the owner of the market then form an alliance of business to generate revenue through online sales. However, the efficiency and stability of the alliance relies on the mechanism (contract) that specifies the way to split the revenue and costs over the members of alliance. Thus we are interested in alliance coordinating contract that gives each party positive expected profit and there is no other contract which gives a better profit to one party without sacrificing the other party's profit. In other words, there is no better contract such that both parties of the alliance are happy to move to. Therefore the concept of coordination specifies the efficiency and stability of an alliance.

We will consider some typical contracts and examine whether they coordinate an online market alliance. One of the simplest and widely used contract in e-business is fixed-fee charging, i.e. the market owner always gets a fixed amount from the revenue regardless the amount of overall revenue and costs of the alliance. We prove that if the costs of online market can be ignored, fixed-fee charging coordinates the alliance.

However, the result is no longer true if the costs of market owner are significant. This result includes an important aspect of the online market. A typical example is to put an advertisement works to increase revenue. Unlike the traditional supply chains, the cost owner of advertisement can be the online market owner instead of the sellers. We show that the property of contracts may change according to the effect of the costs using an advertisement example. Next, we consider another common used contract – revenue sharing, under which each party receives a certain percentage of revenue from the overall revenue. Unfortunately, the contract is unable to coordinate an alliance even if the costs of the market owner are ignorable. Finally we represent *profit sharing contract* that coordinates the online market alliance. Profit sharing contract predetermine a proportion of sharing revenue among the alliance members. Meanwhile, the cost is shared by the opponent's proportion of the revenue. Hence it balances out the revenue and the cost among the alliance members.

As far as we know, few works have tackled this problem in online market. In [1], Netessine et al. consider the problem of coordination for online retailers that behave similar to the traditional supply chains [2, 3]. That is, online retailers buy products and sell them online. In our model, we do not assume this behavior (the online market owner do not buy these products). As a result, our proposal differs from the coordination in the traditional supply chains.

In Section 2, we present the concepts of contract and alliance coordination based on a generic market model. In Section 3, we detail the characteristics of online market. In the next two sections, we formally characterize the properties of fixed-fee contract (Section 4) and revenue-sharing contract (Section 5) in the context of online market. In Section 6, we propose a contract that achieves alliance coordination for online market. Finally, in Section 7, we discuss some related works and conclude the paper.

2 Alliance Coordinating Contracts

As soon as agents commit to a joint action in order to obtain some revenues they have to define the way they will share these revenues. This way is usually detailed in a contract that leads agents to establish an alliance. The aim of a contract is to detail the revenue for each member of the alliance. Knowing the contract, members of the alliance will determine their actions. In the following, we formally describe all these basic concepts.

2.1 Contract in Alliance

Consider an alliance $\mathcal{A}$ in which there are n agents. Each agent i $(1 \leq i \leq n)$ has a strategy space S_i. Let $\mathcal{S} = \prod_{i=1}^{n} S_i$. Each agent i chooses a strategy $\sigma_i \in S_i$. Let $\sigma = (\sigma_1, \cdots, \sigma_n) \in \mathcal{S}$ be a strategy profile and σ_{-i} be the strategy profile for all agents except i. We assume that alliance $\mathcal{A}$ earns an alliance revenue as the result of the strategic choices of all agents in $\mathcal{A}$. Let $R : \mathcal{S} \rightarrow \Re$ be a function representing alliance revenue. In many cases, the strategy is interpreted as investments, efforts, or actions incurring costs to generate revenue. Since the alliance revenue is generated as a result of joint-work among the agent, the way to share the revenue is significant for the alliance members. In general, the way of sharing is specified in a contract defined as follows.

Definition 1. *A contract of an alliance $\mathcal{A}$ is a function $\tau : \mathcal{S} \times \Re \rightarrow \Re^n$ which satisfies $\sum_{i \in \mathcal{A}} \tau_i(\sigma, r) = r$ for any $\sigma \in \mathcal{S}$ and $r = R(\sigma)$.*

That is, w.r.t. an alliance revenue, a contract defines individual revenues for each agent based on the strategic choices. We assume the alliance revenue is non-negative. Each individual revenue may include fees and incentives, hence it may be negative.

Example 1. Consider a joint investment alliance on natural resource development consisting of three investors, $\mathcal{A} = \{a, b, c\}$. Let σ be a set of monetary investments of all alliance members. According to the investments, natural resource $r = R(\sigma)$ is mined. Suppose all members agree on the following contract before the investments: Each share of r is proportional to the individual amount of investment; that is $\tau_i(\sigma, r) = r\left(\sigma_i / \sum_{i \in \mathcal{A}} \sigma_i\right)$ for all i.

The above example is an ideal case, since the profits are shared among the alliance members. However, the profit-sharing is not always effective or implementable in industry. For instance, an individual cost cannot always be associated to specific alliance revenue or cost information is usually private information.

Definition 1 shows that individual revenues are based on strategies and contracts. It means that agents may choose their strategies with respect to the contract. In order to choose the strategies agents need criteria to evaluate their profits in terms of strategies and contracts. Let $\pi_i^\tau(\sigma)$ be the profit of agent i at strategy profile σ and contract τ. Profit is based on revenue and cost. Let $c_i(\sigma_i)$ be the cost function of agent i. Then the profit function is $\pi_i^\tau(\sigma) = \tau_i(\sigma, r) - c_i(\sigma_i)$. We assume that agents choose their own strategies to maximize their own profits. Since the profit of agent i depends on the other agents' strategies, the evaluation of profits is explained by Nash equilibrium. Formally:

Definition 2. *Given alliance $\mathcal{A}$, contract τ, the set of all strategy spaces $\mathcal{S}$, and profits $\{\pi_i^\tau(\sigma)\}_{i \in \mathcal{A}}$, the profile $\hat{\sigma} \equiv (\hat{\sigma}_1, \cdots, \hat{\sigma}_n)$ is called a Nash equilibrium if for all $i \in \mathcal{A}$, and for all $\sigma_i \in S_i$, $\pi_i^\tau(\hat{\sigma}) \geq \pi_i^\tau(\sigma_i, \hat{\sigma}_{-i})$.*

It means that at Nash equilibria, no agent can increase its profit by changing strategies w.r.t. contract τ. However, even if we have a Nash equilibrium, the overall profit may not be maximized. Alliance optimal profit is gained by Pareto optimal contract as follows.

Definition 3. *A contract τ is* Pareto optimal *if there is no other contract τ' such that for any strategy profile $\sigma \in \mathcal{S}$ such that $\pi_i^{\tau'}(\sigma) \geq \pi_i^\tau(\sigma)$ for all i with at least one strict inequality.*

It means that if a contract is Pareto optimal, then there is no other contract where all the agents' profits can increase. Notice that our definition of Pareto optimal contract avoids the case where increasing one agent's profit is possible without decreasing others agents' profits. To investigate Pareto optimal contracts, we have to compare profits for each strategy profile, each contract and each agent. In order to simplify this task, we introduce the notion of alliance optimal profit. Let the alliance profit be $\pi(\sigma) = R(\sigma) - C(\sigma)$, where $C(\sigma)$ is a linear combination of each cost $c_i(\sigma_i)$. Let the alliance optimal profile be $\sigma^* \equiv (\sigma_1^*, \cdots, \sigma_n^*)$ such that $\sigma^* = \arg\max_{\sigma^* \in \mathcal{S}} \pi(\sigma)$. The following proposition shows how to check Pareto optimality of contracts.

Proposition 1. *Let σ^* be an alliance optimal strategy profile and let $\hat{\sigma}^\tau$ be the unique Nash equilibrium under a contract τ. If $\hat{\sigma}^\tau = \sigma^*$, then contract τ is a Pareto optimal.*

Proof. The assumptions of the proposition entail that we have $\sum_{i \in \mathcal{A}} \pi_i^\tau(\hat{\sigma}^\tau) = R(\hat{\sigma}^\tau) - \sum_{i \in \mathcal{A}} c_i(\hat{\sigma}^\tau) = R(\sigma^*) - C(\sigma^*) = \pi(\sigma^*)$. Since a strategy profile σ^* maximizes the alliance profit, there is no other way to increase the alliance profit. Hence, contract τ is Pareto optimal. □

Choosing contract and setting its parameters is a way to give incentive to agents for their participation in an alliance. For instance, contract may guarantee some revenue. This is especially significant if demand has to be considered as uncertain. Taking into account uncertainty of demand is mandatory, since agents have to decide their behavior before they realize the actual demand. Decision criteria of agents are dependent on risk tendencies. Since we assume all the agents are risk neutral, these decisions are only dependent on the expected profits. We denote E as expectation for stochastic variables. Based on the above settings, we define an alliance coordinating contract as follows.

Definition 4. *Given alliance $\mathcal{A}$, contract τ coordinates alliance $\mathcal{A}$, if it satisfies the following conditions,*

1. *contract τ is Pareto optimal,*
2. *there exists a strategy profile σ such that $E[\pi_i^\tau(\sigma)] > 0$ for all $i \in \mathcal{A}$.*

The definition shows that our interested contracts must be acceptable by every agents. That is, in addition to the Pareto optimality, the alliance coordination requires that all expected profits should be positive; this constraint is called participation constraint.

3 Online Market Model

In the previous section, we have shown the definition of contract and the alliance coordinating contract. Let us instantiate this framework in the context of online market. We consider an online market where the sellers and buyers trade items. In this online market, an alliance consists of an online market owner and a seller.

For strategic choices of the two alliance members, we consider the seller chooses the listing quantity of a single type of items on the online market and the online market owner chooses advertisement amount. As mentioned, we suppose that the sales depend on a given stochastic demand at given price. The revenue is shared between the two alliance members according to the contract. The seller directly ships the item to the buyers and thus buyers are not considered as members of the alliance.

Formally, let $\mathcal{A} = \{o, s\}$ be the alliance such that o is the online market owner and s is the seller. The strategy of o is to choose advertisement amount a and the strategy of s is to choose listing quantity q. For a given price p, we assume a random demand $X(a)$ which is affected by advertisement amount a. Let F be the cumulative distribution function of the demand and f be its probability distribution function. If the listing quantity is q and the advertisement amount is a, we obtain the expected revenue $R(q, a) = pQ(q, a)$ where $Q(q, a)$ is the expected sales quantity such that $Q(q, a) = E[\min\{X(a), q\}]$. It means that if listing quantity q is greater than demand, then sales quantity is a demand, otherwise the sale quantity is inventory quantity q. Stochastic demand entails that $E[\min\{X(a), q\}]$ and thus $Q(q, a)$ are equal to:

$$Q(q, a) = \int_0^q xf(x|a)dx + \int_q^\infty qf(x|a)dx. \tag{1}$$

By differentiating $Q(q, a)$ w.r.t. q, we get that the increase of inventory quantity for one unit results in the increase of the expected sales quantity less than one unit (since $\frac{\partial Q(q,a)}{\partial q} = 1 - F(q|a) < 1$). For the advertisement effect, we assume positive effect on the expected sales (that is $\frac{\partial Q(q,a)}{\partial a} \geq 0$). Furthermore, we assume that the expected sales is diminishing concave (i.e. $\frac{\partial^2 Q(q,a)}{\partial a^2} \leq 0$ and $\frac{\partial^3 Q(q,a)}{\partial a^3} \geq 0$).

For online trades, we consider the following costs for online market owner o and seller s. For agent o the cost is equal to the fixed cost $c_{\bar{o}}$ plus advertisement cost. Let $g(a)$ be a cost function for advertisement a where $g(0) = 0$, $\frac{dg(a)}{da} > 0$ and $\frac{d^2 g(a)}{da^2} \geq 0$. In other words, the cost of advertisement and the marginal advertisement cost both increase w.r.t. advertisement amount. For seller s, at the time of listing quantity on the online market, we assume that items are already prepared as inventories with unit cost c_p. We also consider fixed cost for seller s as $c_{\bar{s}}$ and shipment cost c_s per unit. For simplicity, we do not consider either salvage value, stock out penalty, or inventory holding cost.

At the opposite of listing quantity, advertisement can be null to generate revenue. In such a case, the alliance strategy (q, a) is equal to $(q, 0)$ and whenever it is clear q stands for the strategy. Let us detail the case where there is no advertisement. By choosing listing quantity q, the alliance expects to earn revenue $r = R(q) = pQ(q)$ at given price p. Suppose the alliance agrees on a contract τ, the expected profit of online market owner o is

$$E[\pi_o^\tau] = \tau_o(q, r) - c_{\bar{o}} \tag{2}$$

and the expected profit of seller s is

$$E[\pi_s^\tau] = \tau_s(q, r) - c_s Q(q) - c_p q - c_{\bar{s}} \tag{3}$$

These two functions show that the profits of alliance members depend on the choice of listing quantity by the seller and of the contract. Meanwhile, the choice of the listing quantity incurs variable costs for the seller however it does not incurs any variable costs for agent o. This setting is very unique for online market compared to the traditional supply chains [2]. While physical distributions are executed among alliance members that incur variable costs in the traditional supply chains, online market does not incur any variable costs due to direct shipment from the seller to the buyer.

Based on the above settings, we first show that there exists an alliance optimal profit for this model.

Lemma 1. *Let q^* be an alliance optimal listing quantity in the online market model without advertisement effect. There exists a unique optimal listing quantity* $q^* = F^{-1}\left(\frac{p-c_s-c_p}{p-c_s}\right)$ *in the online market model.*

Proof. According to the definition of the profit, we obtain the alliance expected profit as follows, $E[\pi(q)] = (p-c_s)Q(q) - c_p q - (c_{\bar{o}} + c_{\bar{s}})$. The first-order derivative of the expected profit is

$$\frac{dE[\pi(q)]}{dq} = (p-c_s)(1-F(q)) - c_p \tag{4}$$

The second-order derivative of Equation (4) is $\frac{d^2E[\pi(q)]}{dq^2} = -(p-c_s)f(q)$. Since $f(q)$ is positive, we obtain that the alliance profit function is concave in quantity q. Therefore, the alliance optimal quantity q^* must be the solution of Equation (4) such that $q^* = F^{-1}\left(\frac{p-c_s-c_p}{p-c_s}\right)$. □

Lemma 1 shows that the unique alliance optimal quantity exists in this model. Therefore, an equilibrium listing quantity under a certain contract must be equal to q^*. Based on this optimal quantity, we investigate the alliance coordinating contracts in the case of no advertisement . We focus on an interesting case where $E[\pi(q^*)] > 0$.

Now let us relax the assumption of no advertisement. Taking into account advertisement entails to redefine the Pareto optimality checking, since the alliance profit function is different. The alliance expects to earn revenue $r = R(q,a) = pQ(q,a)$ at given price p. Suppose the alliance agrees on a contract τ, the expected profit of agent o is

$$E[\pi_o^\tau(q,a)] = \tau_o(q,a,r) - c_{\bar{o}} - g(a), \tag{5}$$

and the expected profit of agent s is

$$E[\pi_s^\tau(q,a)] = \tau_o(q,a,r) - (1-\alpha)c_s Q(q,a) - c_p q - c_{\bar{s}} \tag{6}$$

In order to check the alliance coordination, we define the expected alliance profit as follows,

$$E[\pi(q,a)] = (p-c_s)Q(q,a) - c_p q - (c_{\bar{o}} + c_{\bar{s}}) - g(a). \tag{7}$$

According to the definitions of Q, Equation (7) is concave in listing quantity q and a. Therefore, there exists an alliance optimal pair $\{q^*, a^*\}$. Notice that this pair is not necessary unique. We suppose that for any given fixed listing quantity q, there exists optimal advertisement amount $a^*(q)$. Formally, this is represented by the following first-order condition, similarly to [4, 3]:

$$\frac{\partial \pi(q,a^*(q))}{\partial a} = (p-c_s)\frac{\partial Q(q,a^*(q))}{\partial a} - \frac{dg(a^*(q))}{da} = 0 \tag{8}$$

For this online market model, we now investigate the properties of alliance coordinating contracts. We focus on two typical contracts: *fixed-fee contract* and *revenue-sharing contract*. For these two contracts, we study the advertisement effect.

4 Fixed-fee Contract

Fixed-fee contract is employed in many online markets. Fixed-fee contract is a contract where one agent always gets the same individual revenue regardless the alliance revenue, formally:

Definition 5. *A contract τ of an alliance $\mathcal{A}$ is called to be fixed-fee contract by agent i_0 if it satisfies the following conditions: for any $\sigma \in \mathcal{S}$ and $r \in \Re$,*

1. $\tau_{i_0}(\sigma, r) = \alpha$
2. $\sum\limits_{i \neq i_0} \tau_i(\sigma, r) = r - \alpha$

where $\alpha \in \Re$ is constant and interpreted as the charging fee.

Under fixed-fee contract, agent i_0 charges fixed-fee α to the other agents and the returns of the alliance is taken by agents except for agent i_0. In the context of online market, we have $\tau_o(\sigma, r) = \alpha$ and $\tau_s(\sigma, r) = r - \alpha$ s.t. $\sigma = (q, a)$. Notice that charging a membership fee is a similar contract.

Example 2. Consider an online market where sellers sell second-hand items to buyers. In this online market, the owner o charges $\$2$ fixed-fee to seller s for each listing. The contract can be represented as follows:

$$\tau_o(\sigma, r) = 2; \qquad \tau_s(\sigma, r) = r - 2.$$

It means for any seller's strategy the market owner's share of revenue is constant. This contract is used in eBay BuyItNow option.

4.1 No Advertisement

The following proposition shows that the online market model without advertisement effect achieves alliance coordination.

Proposition 2. *Let α be a fixed-listing fee of the online market. Fixed-fee contract τ achieves alliance coordination in online market model without advertisement effect, if $c_{\bar{o}} < \alpha < (p - c_s)Q(q^*) - c_p q^* - c_{\bar{s}}$.*

Proof. If $q > 0$, under fixed-fee contract, the expected profit of online market owner o and seller s are respectively,

$$E\left[\pi_o^\tau(q)\right] = \alpha - c_{\bar{o}} \tag{9}$$

$$\begin{aligned} E\left[\pi_s^\tau(q)\right] &= r - \alpha - c_s Q(q) - c_p q - c_{\bar{s}} \\ &= (p - c_s)Q(q) - c_p q - \alpha - c_{\bar{s}}, \end{aligned} \tag{10}$$

otherwise, we have $\pi_o^\tau(q) = \pi_s^\tau(q) = 0$. Since the expected profit of online market owner o is always α if $q > 0$, the online market owner concerns whether the seller lists items at quantity $q > 0$ or not. Hence, we check the optimal listing quantity for seller s denoted as $\hat{q}_s$. By differentiating the profit function of seller s w.r.t. q, we obtain

$\frac{dE[\pi_s^\tau(q)]}{dq} = (p - c_s)(1 - F(q)) - c_p$. According to Equation (4), we get $\frac{dE[\pi_s^\tau(q)]}{dq} = \frac{dE[\pi(q)]}{dq}$ and according to Lemma 1, there is a unique alliance optimal quantity q^* in this model. Thus we obtain $\hat{q}_s = q^*$. For participation constraints, Equation (9) and (10) must be positive at $q = q^*$. Therefore, fixed-fee contract achieves alliance coordination, if fixed-fee α satisfies $c_{\bar{o}} < \alpha < (p - c_s)Q(q^*) - c_p q^* - c_{\bar{s}}$. □

Proposition 2 shows that the alliance coordination is due to the cost structure of the online market owner which does not incur variable cost for the listing quantity. As long as the fee is greater than the owner's cost and lower than the seller's expected profit, the alliance coordination holds.

4.2 Advertisement Effect

According to the advertisement effect, strategies are now pairs (q, a) and the contracts are $\tau_o(q, a, r) = \alpha$ and $\tau_s(q, a, r) = r - \alpha$. The following proposition shows that fixed-fee contract does not achieve alliance coordination.

Proposition 3. *Fixed-fee contract τ does not achieve alliance coordination in the online market model with advertisement effect.*

Proof. Under fixed-fee contract τ, if $q > 0$, according to Equation (5) and the definition of the contract, the profit function of online market owner o is: $E\left[\pi_o^\tau(q, a)\right] = \alpha - c_{\bar{o}} - g(a)$. The first-order derivative of profit function of online market owner o w.r.t. a is $\frac{d\pi_o(a)}{da} = -\frac{dg(a)}{da} < 0$. Hence, under fixed-fee contract, online market owner o does not have incentive to place any positive advertisement amount which is the assumption of the online market model without advertisement effect shown in the previous section. Therefore, fixed-fee contract does not achieve alliance coordination in this model. □

According to Proposition 3, seller s enjoys a benefit of advertisement effect as a free rider under fixed-fee contract. Furthermore, the online market owner does not have any incentive to place advertisement in a context of alliance coordination.

We have shown that fixed-fee contract achieves alliance coordination in the limited case where advertisement is not considered. The next question we address is whether the other popular contract, revenue-sharing contract, achieve alliance coordination.

5 Revenue-Sharing Contract

The following contract, *revenue-sharing contract*, is frequently used in the online markets. Individual revenue is a proportion of the alliance revenue [3].

Definition 6. *A contract τ of an alliance $\mathcal{A}$ is called to be revenue-sharing contract if there exists $\alpha_1, \cdots, \alpha_n$ s.t. $\sum_{i \in \mathcal{I}} \alpha_i = 1$ and for any $(\sigma_1, \cdots, \sigma_n) \in \mathcal{S}$ and $r = R(\sigma)$,*

$$\tau_i(\sigma_1, \cdots, \sigma_n, r) = \alpha_i r \text{ for all } i$$

Example 3. Consider an online music market for selling songs. The alliance consists of online music store o and music label s. The contract specifies the following royalties on revenue r: 20% of r for agent o and 80% for agent s. Hence, the contracts are:

$$\tau_o(\sigma, r) = 0.20r; \qquad \tau_s(\sigma, r) = 0.80r$$

5.1 No Advertisement Effect

Under revenue-sharing contract, the online market owner charges a certain portion of the sales amount of the seller. Portion α ranges in $0 < \alpha < 1$. Hence $\tau_o(q, r) = \alpha r$ and $\tau_s(q, r) = (1 - \alpha)r$. The following proposition shows that this contract does achieve alliance coordination.

Proposition 4. *Let α be a portion that online market owner o earns from the revenue r. Revenue-Sharing contract τ does not achieve alliance coordination in the online market model without advertisement effect.*

Proof. Under revenue-sharing contract, the expected profit of online market owner o is

$$\begin{aligned} E\left[\pi_o^\tau(q)\right] &= \tau_o(q, r) - c_{\bar{o}} \\ &= \alpha p Q(q) - c_{\bar{o}} \end{aligned}$$

and the expected profit of seller s is

$$\begin{aligned} E\left[\pi_s^\tau(q)\right] &= \tau_o(q, r) - c_s Q(q) - c_p q - c_{\bar{s}} \\ &= (1 - \alpha) p Q(q) - c_s Q(q) - c_p q - c_{\bar{s}}. \end{aligned}$$

The first-order condition for the profit maximizing quantity of seller s is $\frac{dE[\pi_s^\tau(q)]}{dq} = ((1 - \alpha)p - c_s)(1 - F(q)) - c_p = 0$. Let $\hat{q}_s$ be the profit maximizing quantity of the seller under revenue-sharing contract. We obtain $\hat{q}_s = F^{-1}\left(\frac{p(1-\alpha) - c_s - c_p}{p(1-\alpha) - c_s}\right) > q^*$. Hence, revenue-sharing contract does not achieve alliance coordination. □

As mentioned, the online market owner does not incur any variable cost or any procurement cost. Therefore, revenue-sharing contract does not achieve alliance coordination. Even though the proposition shows that revenue-sharing contract is not an alliance coordinating contract, it is a popular contract in online market. Parameter α is usually set at a small value in the online markets. Therefore, it entails that seller s may list slightly greater quantities than the alliance optimal quantity. This means that the listed quantity entailed by a revenue-sharing contract may be greater than the one entailed by fixed-fee contract. Therefore, the online market owner may sell greater quantities under revenue-sharing contract compared to fixed-fee contract.

5.2 Advertisement Effect

Let α be the online market owner o's portion of revenue. The contracts are $\tau_o(q, a, r) = \alpha r$ and $\tau_s(q, a, r) = (1 - \alpha)r$. Again we show that revenue-sharing contract does not achieve alliance coordination.

Proposition 5. *Revenue-sharing contract τ does not achieve alliance coordination in the online market model with advertisement effect.*

Proof. Under revenue-sharing contract τ, for a given listing quantity q, let $\hat{a}_o(q)$ be the optimal advertisement amount for online market owner o corresponding to listing quantity q. It entails that the first-order condition represented in Equation (8) holds for $(q, \hat{a}_o(q))$. Hence, for the optimal profit function π_o^τ, we have $\frac{\partial E[\pi_o^\tau(q,\hat{a}_o(q))]}{\partial a} = \alpha(p - c_s)\frac{\partial Q(q,\hat{a}_o)}{\partial a} - \frac{dg(\hat{a}_o)(q)}{da} = 0$. According to Equation (8) , if $\hat{a}_o = a^*$ we have $\frac{\partial E[\pi_o^\tau(q,\hat{a}_o)]}{\partial a} < \frac{\partial E[\pi(q,a^*(q))]}{\partial a}$. Thus we have $\hat{a}_o \neq a^*$. Hence, revenue-sharing contract does not achieve alliance coordination in the case of individual advertisement.

□

According to Propositions 4 and 5, revenue-sharing contract does not achieve the alliance coordination regardless of advertisement effect. This is mainly due to the lack of relation between marginal cost and marginal profit. In the next section, we propose a contract that takes care of this relation.

6 Profit Sharing Contract

The aim of this contract is to balance out revenue and variable costs between alliance members. That is each member does not only consider its cost to define its profit, but also the other members' costs. The revenue is $r = R(\sigma_i, \sigma_j)$. Let $\chi_i > 0$ be a parameter for setting at first the portion of revenue for agent i and, second the portion of cost that agent j charges to agent i. We assume that $\sum_{i \in \mathcal{A}} \chi_i = 1$. The following contract is in the scheme of profit sharing contract in [5],

Definition 7. *Let $\chi_i > 0$ be a portion parameter of profit sharing contract τ. A contract τ of an alliance $\mathcal{A}$ is a profit sharing contract if $\tau_i(\sigma_i, \sigma_j, r) = \chi_i r - \chi_i c_j(\sigma_j) + \chi_j c_i(\sigma_i)$ for all $i \in \mathcal{A}$.*

In the context of online market, profit sharing contract is interpreted as follows:

$$\tau_o(q, a, r) = \chi r - \chi c_s S(q, a) - \chi c_p q + (1 - \chi)g(a)$$
$$\tau_s(q, a, r) = (1 - \chi)r + \chi c_s S(q, a) + \chi c_p q - (1 - \chi)g(a)$$

This profit sharing contract is a combination of revenue-sharing, sales discount, listing incentive and advertisement cost sharing.

The following theorem shows that profit sharing contract achieves alliance coordination.

Theorem 1. *Profit sharing contract τ achieves alliance coordination in the online market model.*

Proof. W.r.t. τ, the profit function for online market owner o is

$$E[\pi_o^\tau(q, a)] = \chi((p - c_s)S(q, a) - c_p q - g(a)) - c_{\bar{o}}, \tag{11}$$

and the profit function of seller s is

$$E\left[\pi_s^\tau(q,a)\right] = (1-\chi)((p-c_s)S(q,a) - c_p q - g(a)) - c_{\bar{s}}. \qquad (12)$$

By differentiating Equation (11) w.r.t. a, we obtain a marginal profit of online market owner w.r.t. advertisement $\frac{\partial E[\pi_o^\tau(q,a)]}{\partial a} = \chi\left((p-c_s)\frac{\partial S(q,a)}{\partial a} - \frac{dg(a)}{da}\right)$. Since $(p-c_s)\frac{\partial S(q,a)}{\partial a} - \frac{dg(a)}{da} = \frac{\partial E[\pi(q,a)]}{\partial a}$, we have $\frac{\partial E[\pi_o^\tau(q,a)]}{\partial a} = \chi\frac{\partial E[\pi(q,a)]}{\partial a}$. Therefore, it satisfies the first-order condition shown in Equation (8). A condition to satisfy participation constraint is Equation (11) and (12) must be positive at a pair $\{q^*, a^*\}$ as follows:

$$E\left[\pi_o^\tau(q^*,a^*)\right] = \chi\left((p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)\right) - c_{\bar{o}} > 0$$
$$E\left[\pi_s^\tau(q^*,a^*)\right] = (1-\chi)\left((p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)\right) - c_{\bar{s}} > 0$$

Therefore, we obtain that profit sharing contract achieves alliance coordination, if

$$\frac{c_{\bar{o}}}{(p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)} < \chi < \frac{(p-c_s)S(q^*,a^*) - c_p q^* - g(a^*) - c_{\bar{s}}}{(p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)}$$

□

Profit sharing contract charges their costs to the alliance partner with prefixed portion each other. At the same time, based on the opposite portion, the revenue is shared. According to Theorem 1, profit sharing contract balances out the costs of the alliance members by sharing them. So far, we are not aware of any online market companies that implement this contract. This is because to obtain cost information from the seller is costly and to reveal online market's cost information to the sellers is too sensitive.

7 Conclusion and Related work

In this paper, we have at first presented the notion of contract and alliance coordination. Next we have shown how this framework can be used to describe a specific kind of market namely online market. Then, we have studied behavior of this market w.r.t two popular contracts: fixed-fee contract and revenue-sharing contract. We have shown that it is difficult to obtain coordination for these two contracts: only fixed-fee contract with no advertisement achieves alliance coordination. Revenue-sharing contract leads sellers to list greater quantities compared to the case of fixed-fee contract; this property may be a desirable one for gaining market shares. We finally exhibit a profit sharing contract that enables to achieve coordination. Even if this contract is difficult to implement, it may be used for setting more general conditions of alliance coordination, since this contract characterizes the aspects of efficiency and stability. For instance, profit sharing can be approximated to revenue-sharing plus fixed-fee in very limited cases. However, profit sharing is able to indicate the revenue-sharing plus fixed-fee contract's parameter settings. This contract is actually implemented by eBay's auction.

Our definition of contract slightly differs from the one given by Gan, et al. in [6]. They define a contract as a proportion of the alliance revenue. Their definition mainly focuses on revenue-sharing contract. Our framework is more suitable for describing different types of contract such as fixed-fee contract.

We have assumed that all agents are risk neutral similar to others settings [2, 3, 1]. Therefore, we consider that the decision making criteria of the agents are their expected profits. If we want to consider risk averse agents, our definition of alliance coordination may be extended for taking it account utilities of agents as proposed in [6, 7].

We remark that there is a significant difference between the concepts of coalition formation in game theory and alliance coordination we have discussed in this paper, though both of them concern about how a group of agents share the gains from co-operation. Coalition games are described in terms of payoffs of coalitions (subgroups) rather than payoffs of individuals [8]. The main concern of a player in a coalition game is which subgroup he/she should join in order to maximize his/her outcomes. In our model, we assume that all agents are in the same alliance, i.e., a grand coalition. The concern of an agent is how much he/she should invest to the coalition to get the maximal return, given a certain coordination contract.

In this model, we did not consider some specific aspects of online markets like non-cooperative shipment which is the most significant problem in eBay like market according to [9]. In order to deal with this problem, it is siginificant to embed the concept of reputation into this model. We set this point as future work.

In online market, a key question is to know how to attract traders. For the online market owners such as eBay or lastminute.com, it involves uncertain and incomplete information. As a consequence, it is not always possible to find the optimal strategies. In order to find the solutions for this complex problem, Trading Agent Competition in Market Design (TAC-MD) has been proposed as a simulation test-bed [10]. We can view the problems of TAC-MD as a coordination problem and thus as future work we want to show how our proposal fits the TAC-MD framework.

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