Consistency of Action Descriptions

Dongmo Zhang[1], Samir Chopra[2], and Norman Foo[2]

[1] School of Computing and Information Technology
University of Western Sydney, Australia
[2] School of Computer Science and Engineering
The University of New South Wales, Australia

Abstract. As a contribution to the metatheory of reasoning about actions, we present some characteristics of the consistency of action theories. Three levels of consistency are investigated for the evaluation of action descriptions: uniform consistency, consistency of formulas and regional consistency. The first two provide an intuitive resolution of problems of explanation conflicts and fluent dependency. The concept of regional consistency provides for a measure of ramification. A highly expressive form of action descriptions, the normal form, is introduced to facilitate this analysis. The relative satisfiability of the situation calculus is generalized to accommodate non-deterministic effects and ramifications.

1 Introduction

The metatheory of logical frameworks for reasoning about actions has received justified attention in recent times [15, 18, 12, 24, 21]. These studies have helped to establish a systematic methodology for the evaluation of the various frameworks proposed for reasoning about action. An important baseline property for all formal systems is *consistency*. In reasoning about actions, an accurate and consistent action description is crucial since problems in the action description infect all further reasoning about the dynamic domain it describes. We show that the issues raised in the consistency analysis of action descriptions are significant and interesting. The consistency of both the logical system itself *and* the action description of the dynamic domain needs to be evaluated. Incorrect, incomplete and inconsistent action descriptions can be detected and rectified, leading to a better understanding of the dynamic domain and a better formalization of the problem.

Several formalisms exist for reasoning about actions such as the situation calculus [23], action languages [6], the event calculus [13], the fluent calculus [26] and dynamic logic [10]. Since consistency is generally defined in terms of the associated deductive system and its properties generally require semantical consideration, a logic of action possessing a sound and complete deductive system would be most helpful in consistency analysis. Seen in this light, dynamic logic might be the best candidate among these formalisms of action. Several advantages to reasoning about actions in dynamic logic can be emphasized. Firstly, dynamic logic, with its underlying semantics of transition systems, is a *natural* framework for reasoning about actions. Dynamic logic was originally developed for reasoning about programs; any program can be viewed as an action and any action can be implemented by programs. Secondly, dynamic logic can specify the entire spectrum of actions: compound, non-deterministic and concurrent. Often, dynamic logic expresses such actions more naturally than other action formalisms. Thirdly, dynamic logic provides a sound and complete axiomatic deductive system and a well-developed Kripkean semantics. Its proof and model theory have reached a high degree of sophistication and maturity. Features such as decidability and the finite model property of propositional dynamic logic (PDL), and techniques such as bisimulation and filtration, are well understood. Within Artificial Intelligence, dynamic logic has been used to investigate computational properties of formalisms such as features logics, description logics and conditional logics. With this in mind, we exploit an extended propositional dynamic logic, $EPDL$ [27]. This system offers a unified treatment of reasoning about direct and indirect effects of actions, thus enabling a representation of action effects and causal ramifications. We introduce techniques for consistency analysis of action descriptions in $EPDL$ frameworks. Three different levels of consistency are provided: *uniform consistency* of action descriptions, *Σ-consistency* of formulas and *regional consistency* of action descriptions. Uniform consistency conveys information about what kinds of action descriptions guarantee proper runs of a dynamic system. Σ-consistency of formulas informs us of which situations a dynamic system can start up from and run properly. It also serves as a tool with which to detect incorrect and inadequate action descriptions. The concept of regional consistency provides for a measure of ramification. Addressing the issue of consistency of action descriptions provides an alternative approach to thinking about classical problems in reasoning about action.

2 EPDL Preliminaries

We summarize some basic facts of the extended propositional dynamic logic ($EPDL$) (see [27] for more details). In propositional dynamic logic (PDL), a causal relation between an action α (primitive or compound) and a property A is expressed by the modal formula: $[\alpha]A$, meaning "α (always) causes A if α is feasible". For example, $[Turn\_off]\neg light$ says that "turning off the switch causes the light to be off". The dual operator $\langle\alpha\rangle A$, reads "α is feasible and may (or possibly) cause(s) A to be true". For instance, $\langle Spin\rangle\neg loaded$ says that "spinning a gun barrel may cause it to be unloaded". $\langle\alpha\rangle\top$ means "α is feasible or executable". In $EPDL$, propositions are allowed as modalities. The formula $[\varphi]A$, termed *propositional causation*, represents a cause-effect relationship between the proposition φ and the formula A and is read as "φ **causes** A". For example, $[short\text{-}circuit]damaged$ says that "short-circuits cause the circuit to be damaged".

A language $\mathcal{L}_{EPDL}$ of $EPDL$ consists of a set $\mathbf{Flu}$ of fluent symbols and a set $\mathbf{Act}_P$ of primitive action symbols. Propositions ($\varphi \in$ **Pro**), formulas ($A \in$ **Fma**) and actions ($\alpha \in$ **Act**) are defined by the following BNF rules:

$\varphi ::= f \mid \neg\varphi \mid \varphi_1 \rightarrow \varphi_2$
$A ::= f \mid \neg A \mid A_1 \rightarrow A_2 \mid [\alpha]A \mid [\varphi]A$
$\alpha ::= a \mid \alpha_1;\alpha_2 \mid \alpha_1 \cup \alpha_2 \mid \alpha^* \mid A?$
where $f \in$ **Flu** and $a \in \mathbf{Act_P}$.

The definitions of $\top$(**true**), $\bot$(**false**), $\vee$, $\wedge$, $\leftrightarrow$ are as usual. A literal is a fluent or its negation. The set of all the literals is denoted by $\mathbf{Flu_L}$. We introduce the following notation:

$\langle[\alpha]\rangle A =_{def} \langle\alpha\rangle\top \wedge [\alpha]A$, meaning "$\alpha$ *must cause* A";
$\prec \alpha \succ A =_{def} \langle\alpha\rangle\top \rightarrow \langle\alpha\rangle A$, meaning "*if* α *is feasible,* α *may cause A*".[3]

The semantics for $\mathcal{L}_{EPDL}$ is similar to PDL. Since the propositional modality $[\varphi]$ is treated as a normal modal operator, the semantic conditions for propositional modalities are exactly the same as action modalities except for the following extra conditions:

- *If* $M \models_w \varphi$*, then* $(w, w) \in R_\varphi$.
- *If* $\models \varphi \leftrightarrow \psi$*, then* $R_\varphi = R_\psi$.

The axiom system for $EPDL$ extends PDL [9] by one axiom:

- CW axiom: $[\varphi]A \rightarrow (\varphi \rightarrow A)$

and one inference rule:

- CE: From $\varphi \leftrightarrow \psi$ infer $[\psi]A \leftrightarrow [\varphi]A$

The classical K axiom and inference rule N (necessitation) are respectively extended to accommodate propositional modalities:

- EK axiom: $[\gamma](A \rightarrow B) \rightarrow ([\gamma]A \rightarrow [\gamma]B)$
- EN: *From A infer* $[\gamma]A$.
  where $\varphi, \psi \in$ **Pro**, $A \in$ **Fma** and $\gamma \in$ **Pro** $\cup$ **Act**.

A formula A is *provable* from a set Γ of formulas, denoted by $\Gamma \vdash A$, if there exist $A_1, \cdots, A_n \in \Gamma$ such that $\vdash (A_1 \wedge \cdots \wedge A_n) \rightarrow A$. Γ is *consistent* in $EPDL$ if $\Gamma \not\vdash \bot$.

3 Action descriptions and their normal forms

$EPDL$ provides a formal language via *action descriptions* to describe the behavior and internal relationships of a dynamic system. These specify the effects and feasibility of actions, causal ramifications and other domain constraints.

Example 1. Consider the Yale Shooting Problem [11]. Let $\mathbf{Flu} = \{alive, loaded, walking\}$ and $\mathbf{Act_P} = \{Load, Shoot, Wait\}$. This problem can be specified by the following action description:

$$\Sigma = \left\{ \begin{array}{l} \neg loaded \rightarrow [Load]loaded \\ loaded \rightarrow [Shoot]\neg alive \\ loaded \rightarrow [Shoot]\neg loaded \\ [\neg alive]\neg walking \\ \langle Load\rangle\top, \langle Wait\rangle\top, \langle Shoot\rangle\top \end{array} \right\}$$

[3] Note that these two operators are dual, i.e., $\prec \alpha \succ A = \neg\langle[\alpha]\rangle\neg A$.

Formulas in an action description are significantly different from ordinary formulas. For instance, the sentence "$loaded \rightarrow [Shoot]\neg alive$" says that whenever $loaded$ is true, $Shoot$ causes $\neg alive$. In the language of situation calculus, this is written as $\forall s(loaded(s) \rightarrow \neg alive(do(Shoot, s)))$. Indeed, we need to view the action description of a dynamic domain as a set of extra axioms (domain axioms in the situation calculus [23]) rather than an ordinary set of formulas in reasoning about the domain.

Definition 1. [27] *Let Σ be an action description. A formula A is Σ-provable, written as $\vdash^{\Sigma} A$, if it belongs to the least set of formulas which contains all the theorems of $EPDL$, all elements of Σ, and is closed under Modus Ponens and EN.*

If Γ is a set of formulas, then $\Gamma \vdash^{\Sigma} A$ means there exists $A_1, \cdots, A_n \in \Gamma$ such that $\vdash^{\Sigma} (A_1 \wedge \cdots \wedge A_n) \rightarrow A$.

Example 2. Consider the action description in Example 1. We can easily prove that

1. $\neg loaded \vdash^{\Sigma} [Load; Shoot]\neg alive$
2. $\neg loaded \vdash^{\Sigma} [Load; Shoot]\neg walking$
3. $\vdash^{\Sigma} \langle [Load; Wait;$ **if** $\neg loaded?$ **do** $Load$ **endif**$; Shoot]\rangle \neg alive$

Note that the action description in Example 1 does not completely specify the domain since it does not include information about *unaffected* fluents. Without frame axioms we can neither prove nor refute the very intuitive relation: $\neg loaded \vdash^{\Sigma} [Load;Wait;Shoot]\neg alive$. A solution to the frame problem then, is necessary for reasoning with such incomplete action descriptions.[4].

Let Σ be an action description. A model M of $EPDL$ is a *Σ-model* if $M \models B$ for any $B \in \Sigma$. It can be proved that if Σ is finite, then *A is Σ-provable iff A is valid in every Σ-model* [27].

3.1 Normal action descriptions

An action description can be any set of formulas in the $EPDL$ language. However, in most cases we prefer the simple normal form in order to obtain better properties and more convenient treatment. The following kinds of formulas are said to be in *normal form*:

- $[\varphi]L$ (causal law)
- $\varphi \rightarrow [a]L$ (deterministic action law)
- $\varphi \rightarrow \prec a \succ L$ (non-deterministic action law)
- $\varphi \rightarrow \langle a\rangle\top$ (qualification law).

  where φ is a propositional formula, L is a literal and a is a primitive action.

[4] Since the frame problem is not the main concern of this paper, we omit a solution for it; we just add frame axioms when needed. See [1, 7, 5, 22, 4] for PDL-based solutions to the frame problem.

An action description Σ is *normal* if each formula in Σ is in normal form. It is easy to see that the action descriptions in Example 1, 3, 5 and 7 are normal. Action description Σ_1 in Example 6 is normal but Σ_2 is not.

Although the normal form is restricted, it is quite expressive. It can express *direct* or *indirect*, *deterministic* or *non-deterministic* effects of actions, and *qualifications* of actions. Most normal forms in other action theories can be transformed into $EPDL$ normal form (propositional case only). For instance, action descriptions written in the form of pre-condition axioms and successor state axioms in the *propositional* situation calculus language (i.e., there are no sorts *object* and function symbols in the language [23]) can be translated into the $EPDL$ normal form by the following procedure:

1. For each pre-condition axiom $Poss(a, s) \equiv \varphi(s)$, the associated laws are:
   $\varphi \to \langle a \rangle \top$, $\neg\varphi \to [a]f$, $\neg\varphi \to [a]\neg f$
   where f can be any fluent symbol (choosing one).
2. For each successor state axiom $f(do(\pi, s)) \equiv \varphi(\pi, s)$, where π is an action variable, the associated laws are:
   $\varphi \to [a]f$, $\neg\varphi \to [a]\neg f$
   where π is instantiated by each primitive action a.

Most components of action languages [6] can also be expressed by $EPDL$ normal form. For example, "a **causes** L **if** φ" in the action language $\mathcal{A}$ can be translated to "$\varphi \to [a]L$"; a static law "**caused** L **if** φ" in the language $\mathcal{C}$ is translated to "$[\varphi]L$"; and an expression "a **may cause** L **if** φ" in $\mathcal{C}$ is translated to "$\varphi \to\prec a \succ L$". The same translation procedure will work for action descriptions in STRIPS [3].

4 Consistency of Action Descriptions

As noted above, an action description acts as an axiomatic specification of a dynamic system highlighting the importance of consistency. We now consider three different levels of consistency: *consistency of formulas*, *consistency of action descriptions* and *consistency of formulas with action descriptions*. Each of these conveys different information about the dynamic system under consideration.

4.1 Uniform consistency of action descriptions

As defined above, a set Γ of formulas is consistent if $\Gamma \nvdash \bot$. Semantically, it means that there is a model in which Γ is satisfied in *some* world. As far as the consistency of an action description is concerned, however, ordinary consistency is not enough to guarantee that a dynamic system runs properly. As a set of domain axioms, an action description should be consistent with *any possible evolution of the dynamic system under any combination of actions*. With this in mind, we define the consistency of action descriptions as follows:

Definition 2. Let Σ be a set of formulas. Σ is *uniformly consistent* if $\nvdash^{\Sigma} \bot$.

By the soundness and completeness of Σ-provability [27], we have

Theorem 1. *Σ is uniformly consistent if and only if there exists a Σ-model.*

Obviously, uniform consistency implies ordinary consistency. The following highlights the difference between the two.

Example 3. Let $\mathbf{Flu} = \{f_1, f_2, f_3\}$ and $\mathbf{Act} = \{a\}$. $\Sigma = \{\langle a\rangle\top$, $[a]f_1$, $[a]f_2$, $f_1 \rightarrow [a]f_3$, $f_2 \rightarrow [a]\neg f_3\}$. Then Σ is consistent but not uniformly so.

1. $\vdash^{\Sigma} [a]f_1 \wedge [a]f_2$	(AD)
2. $\vdash^{\Sigma} f_1 \rightarrow [a]f_3$	(AD)
3. $\vdash^{\Sigma} f_2 \rightarrow [a]\neg f_3$	(AD)
4. $\vdash^{\Sigma} [a](f_1 \rightarrow [a]f_3)$	(2 and EN)
5. $\vdash^{\Sigma} [a]f_1 \rightarrow [a][a]f_3$	(4 and EK)
6. $\vdash^{\Sigma} [a][a]f_3$	(1 and 5)
7. $\vdash^{\Sigma} [a][a]\neg f_3$	(Similar to 6)
8. $\vdash^{\Sigma} [a][a]\bot$	(6 and 7)
9. $\vdash^{\Sigma} \langle a\rangle\top$	(AD)
10. $\vdash^{\Sigma} [a]\langle a\rangle\top$	(9 and EN)
11. $\vdash^{\Sigma} [a]\neg[a]\bot$	(10)
12.$\vdash^{\Sigma} [a]\bot$	(8 and 11)
13. $\vdash^{\Sigma} \bot$	(9 and 12)

Where AD denotes "action description".

By the finite model property of $EPDL$, the uniform consistency of an action description is decidable. However, satisfiability in $EPDL$ is $EXPTIME$-hard. So deciding the consistency of action descriptions is, in general, intractable. Can we put any syntactical restrictions on action descriptions, say normal form, to make it easier? Is any action description in normal form uniformly consistent? Unfortunately, Example 3 shows that this is not true. Further assumptions are necessary.

Let Σ be a normal action description. For any fluent f and any primitive action a, if we merge the action laws about a and $f(\neg f)$ in each form together, there are at most five laws about a and f in Σ:

$$\varphi \rightarrow \langle a\rangle\top$$
$$\varphi_{1,1} \rightarrow [a]f,\ \varphi_{1,2} \rightarrow [a]\neg f$$
$$\varphi_{2,1} \rightarrow\prec a \succ \neg f,\ \varphi_{2,2} \rightarrow\prec a \succ f$$

If φ, $\varphi_{1,1}$and $\varphi_{1,2}$ are true simultaneously, then the action description will contain a contradiction. Similarly for φ, $\varphi_{1,j}$ and $\varphi_{2,j}$ ($j = 1$ or $j = 2$). For simplicity, we make the following assumption.
Assumption 1: $\vdash \neg\varphi \vee \neg\varphi_{1,1} \vee \neg\varphi_{1,2}$ *and* $\vdash \neg\varphi \vee \neg\varphi_{1,j} \vee \neg\varphi_{2,j}$ *($j = 1, 2$).*

If some law is absent, say $\varphi_{1,1} \rightarrow [a]f$, we use $\bot \rightarrow [a]f$ instead. Note that if φ is a proposition, then $\vdash \varphi$ in $EPDL$ if and only if φ is a tautology in the classical propositional logic.

The assumption 1 only acts on action laws. Similar assumptions could be also made about causal laws. An effect of an action can be either a direct effect (caused by an action) or an indirect effect (caused by other propositions). In most cases (but not all), we can separate the indirectly affected fluents from the directly affected ones [14].

Assumption 2: *There is a partition* $\{\mathbf{Flu}_d, \mathbf{Flu}_i\}$ *of* $\mathbf{Flu}$ *such that* $\mathbf{Flu} = \mathbf{Flu}_d \cup \mathbf{Flu}_i$ *and*

1. for each $f \in \mathbf{Flu}_i$*, if both* $[\varphi_1]f$ *and* $[\varphi_2]\neg f$ *are in* Σ*, then* $\vdash \neg\varphi_1 \vee \neg\varphi_2$.

2. for each causal law $[\varphi]L$*, all the fluents in* φ *are from* $\mathbf{Flu}_d$ *and* L *is a literal in* $\mathbf{Flu}_i$*;*

The first condition of the assumption is similar to the assumption 1. The second condition is intended to avoid recursive indirect effects of actions. For the sake of simplicity, we only allow two layers of causal propagation (as in [14]). More complicated cases can be investigated by using the approach in [2].

Definition 3. A normal action description is *safe* if it is satisfies the Assumptions 1 and 2.

It is easy to see that the action descriptions in Example 1 and 5 are safe but 3 and 7 are not. An interesting observation is any action description which is translated from an action description in the propositional situation calculus is *innately* safe.

Proposition 1. *If* Σ *is an action description generated by the procedure in Section 3.1 from a set of precondition axioms and successor state axioms in the propositional situation calculus, then* Σ *is safe.*

The following theorem is one of the main results in the paper.

Theorem 2. *Let* Σ *be a normal action description. If it is safe then is uniformly consistent.*

This theorem gives us a sufficient condition to check the consistency of an action description by using only propositional logic and the syntax of the action description. Therefore if an action description is written in normal form the consistency checking of the action description becomes a co-NP problem. Specially, as a corollary of the theorem and Proposition 1, an action description is innately uniformly consistent if it is written as a set of precondition axioms and successor state axioms. Note that precondition axioms and successor state axioms in propositional situation calculus language are much less expressive than normal form.

4.2 Σ-consistency of formulas

As defined, a set of formulas is consistent if a contradiction cannot be derived from it. More precisely, its consistency means that it is consistent with the basic axioms and inference rules of $EPDL$, which does not guarantee that it is consistent with arbitrary action descriptions.

Let Σ be an action description. A set Γ of formulas is Σ*-consistent* if $\Gamma \not\vdash^{\Sigma} \bot$. It is easy to see that Σ-consistency of Γ requires the consistency of Γ, the uniform consistency of Σ, and even more. For example, $\{loaded, \neg alive, walking\}$ is consistent, but not Σ-consistent, where Σ is the action description in Example 1, which is uniformly consistent. Σ-consistency of a set of formulas conveys the information that a dynamic system can properly run from an initial situation as specified by the formulas. More interestingly, we notice that in classical logic, a set's inconsistency is due to the set itself if the deductive system of the logic is consistent. Σ-inconsistency of a set, is however, due to both the set *and* the action description. If the set consists of observed facts, the inconsistency must lie in the action description. This provides us with a formal tool to detect incorrect or inadequate action descriptions.

Example 4. Consider the Yale shooting scenario with a new action $Entice$ and add the following action law and qualification law (c.f. [26]):

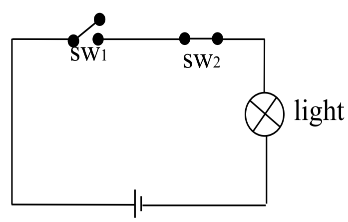
$\neg walking \rightarrow [Entice]walking$
$\neg alive \rightarrow [Entice]\neg alive$
$\langle Entice\rangle\top$

Putting these together with the action description in Example 1 generates a new action description Σ'. Then Σ' is still safe and so uniformly consistent. Note that the set $\{\neg alive, \neg walking\}$ is Σ'-inconsistent.

We can easily see that there is no any problem with the set $\{\neg alive, \neg walking\}$ (these can be observed facts). The problem here can only lie in the action description and specifically, in the newly introduced action laws. Indeed, the qualification law $\langle Entice\rangle\top$ is problematic. A correct description of the qualification of $Entice$ would be: $alive \rightarrow \langle Entice\rangle\top$. We might be tempted to think that this consistency check provides a solution to the qualification problem: we can automatically generate qualification laws from a given action description by default reasoning instead of explicitly listing them in the action description. Unfortunately, this does not always work.

Example 5. Consider the following circuit introduced by [26] and its action description:

$$\Sigma = \left\{ \begin{array}{l} [sw_1 \wedge sw_2]\, light \\ [\neg sw_1 \vee \neg sw_2]\, \neg light \\ \neg sw_i \rightarrow [Toggle_i]sw_i \\ sw_i \rightarrow [Toggle_i]\neg sw_i \\ \langle Toggle_i\rangle\top \\ i = 1, 2 \end{array} \right\}$$



The first sentence says that switch 1 and switch 2 being closed causes the light to be on. The second says that one of the switches being open causes the light to be off. $\neg sw_i \rightarrow [Toggle_i]sw_i$ means that if switch i is open, then toggling switch i causes it to be on. Suppose now that we have an action Hit_the_Bulb. The action laws about the action are:

$light \rightarrow [Hit\_the\_Bulb]\neg light$
$\langle Hit\_the\_Bulb\rangle\top$

Adding these as well as the frame axioms $sw_1 \rightarrow [Hit\_the\_Bulb]sw_1$ and $sw_2 \rightarrow [Hit\_the\_Bulb]sw_2$ to Σ , results in an action description, Σ', which is uniformly consistent. Notice that $\{sw_1, sw_2, light\}$ is Σ'-inconsistent, which is obviously unacceptable.

In this case, it is not reasonable to change the qualification law $\langle Hit\_the\_Bulb\rangle\top$ into $\neg sw_1 \vee \neg sw_2 \rightarrow \langle Hit\_the\_Bulb\rangle\top$. The problem is now in the causal law $[sw_1 \wedge sw_2]light$. We term this the *qualification problem of effect propagation* (for a similar discussion see [17]). The examples above have shown that consistency checks can help us detect incorrect action descriptions. The Stolen Car Problem [25] shows that Σ-inconsistency can be due to the *inadequacy* of the action description.

Example 6. Consider the following action description:

$$\Sigma_1 = \left\{ \begin{array}{l} in\_park \rightarrow [Wait]in\_park \\ \neg in\_park \rightarrow [Wait]\neg in\_park \\ \langle Wait\rangle\top \end{array} \right\}$$

Σ_1 says that waiting does not affect the state of a parked car. It is easy to see that $\{in\_park, [Wait]\ \neg in\_park\}$ is Σ_1-inconsistent. However, the observed facts are exactly that originally the car was parked (in_park) and that it is not there after a period of time ($[Wait]\neg in\_park$).

The problem here is that the agent with this action description has no idea about car's theft: presumably, it should realize that leaving a car alone might cause it to be stolen ($\prec Wait \succ stolen$). A car's theft means that it had been parked somewhere, but disappeared after a period of time. ($in\_park \rightarrow [Wait]\ (\neg in\_park \leftrightarrow stolen)$). So the correct action description should be:

$$\Sigma_2 = \left\{ \begin{array}{l} \prec Wait \succ stolen \\ in\_park \rightarrow [Wait](\neg inpark \leftrightarrow stolen) \\ \langle Wait\rangle\top \end{array} \right\}$$

where $stolen$ is a fluent. Then we have an explanation for the observed facts:

$\{in\_park, [Wait]\neg in\_park\} \vdash^{\Sigma_2} [Wait]stolen.$

We would like to remark that consistency checking can help us detect the incorrectness and inadequacy of an action description but it can not remedy the action description because they are actually two types of problems.

The following theorem is quite useful in the diagnosis of Σ-consistency:

Theorem 3. *Let Σ be a normal and safe action description. Let $D(\Sigma) = \{\varphi \rightarrow L : [\varphi]L \in \Sigma\}$. For any set Γ of propositional formulas, if $\Gamma \cup D(\Sigma)$ is consistent, then Γ is Σ-consistent.*

Therefore, we can check the Σ-consistency of a set of propositional formulas using propositional logic (the complexity of which is in **NP** $\cup$ co-**NP**).

Let us compare the result above with a similar meta-theorem in the situation calculus [21]. Suppose that Σ consists of pre-condition axioms and successor state axioms, and Γ consists of initial state axioms as in the situation calculus. According to Theorem 3 and Proposition 1, Γ is Σ-consistent if and only if Γ is consistent in propositional logic (note that $D(\Sigma)$ is empty here). This coincides with the *Relative Satisfiability* theorem (Theorem 1 in [21]), which says that an action theory $\mathcal{D}$ is satisfiable iff the initial state axioms and unique name axioms are satisfiable. In other words, the foundational axioms, pre-condition axioms and successor state axioms cannot introduce inconsistency. Since the situation calculus in [21] applies to only domains without non-deterministic actions and ramifications, Theorem 3 can be viewed as a generalization of the *Relative Satisfiability* theorem[5].

4.3 Regional consistency of action descriptions

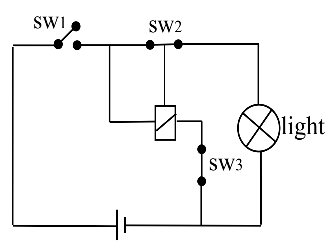
Ramification in dynamic systems arises as a consequence of fluent dependencies. The following notion of consistency provides for a means of assessing the fluent dependencies present in a system.

Definition 4. Let Σ be an action description and U be a subset of $\mathbf{Flu}$. Σ is *regionally consistent* over U if any interpretation I of U is Σ-consistent[6]. Σ is universally consistent if it is regionally consistent over $\mathbf{Flu}$.

Regional consistency of action descriptions reflects local independence of fluents. In other words, if Σ is regionally consistent over U, any change of truth-value of fluents in U does not affect each other (but does affect the fluents outside U). This information is computationally important because once the value of a fluent in U is changed, only the fluents outside U need to be revaluated (see [8]).

Example 7. Consider the circuit introduced by [26] and described with the following simplified action description

$$\Sigma = \left\{ \begin{array}{l} \neg sw_i \rightarrow [Toggle_i] sw_i \\ sw_i \rightarrow [Toggle_i] \neg sw_i \\ [sw_1 \wedge sw_2]\, light \\ [\neg sw_1 \vee \neg sw_2]\, \neg light \\ [sw_1 \wedge sw_3]\, \neg sw_2 \\ \langle Toggle_i \rangle \top \\ i = 1, 2, 3 \end{array} \right\}$$



Then Σ is regionally consistent over $\{sw_1, sw_3\}$, but not over $\{sw_1, sw_2, sw_3\}$ or any supersets. This implies Switch 1 and Switch 3 can be controlled independently, but Switch 2 cannot. So if we take an action $toggle_1$, only those facts which are relevant to the direct effect (sw_1) and the indirect effects (sw_2 and $light$) need to be revaluated (sw_3 can be ignored).

Regional consistency acts also as a *measure* of ramification. The larger the consistent area of an action description, the less ramification it has. If an action description is universally consistent, there is *no* ramification between fluents.

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[5] There are extended versions of situation calculus in the literature [19, 16] which can deal with non-deterministic or indirect effects of actions expressed by successor state axioms. However, Relative Satisfiability is not necessarily true in the extended frameworks without introducing extra restrictions on action descriptions.

[6] An interpretation I of U means a maximal consistent set of literals over U

Proposition 2. *Let Σ be a normal and safe action description. $\{\mathbf{Flu}_d, \mathbf{Flu}_i\}$ is a partition of $\mathbf{Flu}$ which satisfies Assumption 2 of safety. Then Σ is reginally consistent over $\mathbf{Flu}_d$. If there are no causal laws in Σ, then Σ is universally consistent.*

As noted previously, any action description which is translated from a set of precondition axioms and successor state axioms in the propositional situation calculus is universally consistent. This explains why the solution for the frame problem in [23] applies only to actions without ramifications.

The idea of regional consistency is close to the one of *frames in the space of situations*[14]. A frame is a set of fluents which are directly affected by actions. With the concept, the values of the frame fluents can be specified by effect axioms and the law of inertia while the values of non-frame fluents are determined by domain constraints or causal laws. It has been remarked in [14] that a frame be neither too large nor too small. However, it is not clear that what kind of sets of fluents are qualified to be a frame. For the case of normal and safe action descriptions, it is obvious that $\mathbf{Flu}_d$ is a "qualified" frame. For the general case, it is still an open problem. We believe that regional consistency is helpful towards a solution to the problem.

5 Conclusion

In this study we have investigated the characteristics of the consistency of action theory. Three levels of consistency were introduced for the evaluation of action descriptions. These provide an intuitive resolution of problems of explanation conflicts, fluent dependency and a measure of ramification. The highly expressive normal form of action descriptions greatly facilitates such an analysis. Several meta-theorems on the consistency of normal action descriptions have been given which show how to generate a consistent action description and how to check the consistency of normal action descriptions. Our results generalize the Relative Satisfiability Theorem in the situation calculus to allow non-deterministic effects of actions and ramifications. Our study, then, contributes significantly to the meta-theory of reasoning about actions in providing tools for evaluating formally, the adequacy of a logical framework. Although our approach is based on the extended propositional dynamic logic (for its unified expression of direct and indirect effects of actions and its sound and complete deductive system), all the results on the consistency of action descriptions are applicable to other formalisms of actions since the expressions of action descriptions are often intertranslatable. The application of these techniques also leads to new insights on classical problems in reasoning about actions.

Appendix: Proof of Theorems:

Proof of Theorem 2: Let Σ^* be a variant of Σ which is generated by the following procedure:

Step 1: Set $\Sigma = \Sigma^*$ and for each primitive action a,

1. if there is no a qualification law $\varphi \rightarrow \langle a \rangle \top \in \Sigma$, then let $\bot \rightarrow \langle a \rangle \top \in \Sigma^*$;

2. for each fluent literal L, if there is no deterministic action law $\varphi \to [a]L$ in Σ, add $\bot \to [a]L$ to Σ^*.
3. for each fluent literal L, if there is no non-deterministic action law $\varphi \to\prec a \succ L$ in Σ, add $\bot \to\prec a \succ L$ to Σ^*.

Step 2: for each primitive action a and fluent literal L, suppose that all the action laws in Σ^* about a and L are:
$\varphi \to \langle a \rangle \top$
$\varphi_{1,1} \to [a]L$, $\varphi_{1,2} \to [a]\neg L$
$\varphi_{2,1} \to\prec a \succ \neg L$, $\varphi_{2,2} \to\prec a \succ L$
then, we replace $\varphi_{2,1} \to\prec a \succ L$ by $(\neg\varphi \vee \neg\varphi_{1,1}) \to\prec a \succ L$, and $\varphi_{2,2} \to\prec a \succ L$ by $(\neg\varphi \vee \neg\varphi_{1,2}) \to\prec a \succ L$.
We term Σ^* the completion of Σ. It is easy to verify that

1. if Σ is normal and safe, then Σ^* is;
2. $\Sigma_i = \Sigma_i^*$;
3. if Σ^* is uniformly consistent, then Σ is.

Without loss of generality, suppose that $\Sigma = \Sigma^*$. For such an action description Σ, we construct a standard model $M = (W, \{R_\alpha : \alpha \in \mathbf{Act}\} \cup \{R_\varphi : \varphi \in \mathbf{Fma_P}\}, V)$ of $\mathcal{L}_{EPDL}$ as follows:

1. $W = \{w : w$ is an interpretation of Flu and for each $[\varphi]L \in \Sigma$, $w \models_{PL} \varphi \to L\}$. Here $s \models_{PL} \varphi$ means φ is true under the interpretation s by means of propositional logic.
2. For each primitive action $a \in \mathbf{Act_P}$, $(w, w') \in R_a$ iff
   – there exists $\varphi \to \langle a \rangle \top \in \Sigma$ such that $w \models_{PL} \varphi$,
   – for every $\varphi \to [a]L \in \Sigma$, if $w \models_{PL} \varphi$, then $w' \models_{PL} L$; and
   – there exists $\varphi \to\prec a \succ L \in \Sigma$ such that $w \models_{PL} \varphi$ and $w' \models_{PL} L$.
3. For any propositional formula $\varphi \in \mathbf{Fma_P}$, $(w, w') \in R_\varphi$iff $w = w'$ and $w \models_{PL} \varphi$;
4. For any compound action $\alpha \in \mathbf{Act}$, R_α is given inductively by the standard model condition on α.
5. For any primitive proposition p, $V(p) = \{w : w \models_{PL} p\}$.

Let $\Sigma^{PL} = \{\varphi \to L : \exists[\varphi]L \in \Sigma\}$. It is easy to see that M exists if Σ^{PL} is consistent. In fact, Σ^{PL} is consistent because, otherwise, for any interpretation I of $\mathbf{Flu}$, there exists a causal law $[\varphi]L \in \Sigma$ such that $I \not\models_{PL} \varphi \to L$. Pick up such a law $[\varphi]L \in \Sigma$. Then $I \models \varphi \wedge \neg L$. Let I' be the interpretation of $\mathbf{Flu}$ which differs from I only in the interpretation of L i.e., $I' \models L$, or $I' \models \varphi \to L$. If there is no other causal law $[\varphi']\overline{L}$ in Σ , where $\overline{L}$ is the dual literal of L, the truth-values of the formulas in Σ^{PL} other than $\varphi \to L$ stay unchanged under the interpretation I' according to assumption 2 of safety. If there exists $[\varphi']\overline{L} \in \Sigma$, then for $I \models \varphi$, $I \models \neg\varphi'$ (by the assumption 2 of safety again). It follows that $I' \models \neg\varphi'$, or $I' \models \varphi' \to \overline{L}$. Therefore, the number of formulas in Σ which is falsified by I' is one less than the number by I. Continuing this way, we can generate an interpretation which satisfies Σ^{PL}. This means Σ^{PL} is consistent.

It is easy to show that for any $\varphi \in \mathbf{Fma_P}$, $M \models_w \varphi$ iff $w \models_{PL} \varphi$. Now we prove that Σ is valid in M.

1. Suppose that $[\varphi]L \in \Sigma$. For any $w \in W$, according to the construction of W, $w \models_{PL} \varphi \to L$. If $w' \in W$ with $wR_\varphi w'$, by the construction of R_φ, $w = w'$ and $w \models_{PL} \varphi$, so $w \models_{PL} L$. That means $\forall w \in W(M \models_w [\varphi]L)$, so $M \models [\varphi]L$.

2. Suppose that $\varphi \to [a]L \in \Sigma$. For any $w \in W$, if $M \models_w \varphi$, then $w \models_{PL} \varphi$. Thus for any $w' \in W$ with $wR_a w'$, by the construction of M, $w' \models_{PL} L$, so $M \models_{w'} L$. Therefore $M \models \varphi \to [a]L$.

3. Suppose that $\varphi_0 \to \prec a \succ L_0 \in \Sigma$. Let $\varphi_1 \to \langle a \rangle \top$ be the qualification law for a. For any $w \in W$, if $M \not\models_w \varphi_1$, then $w \not\models_{PL} \varphi_1$. According to the construction of M, there is no $w' \in W$ such that $(w, w') \in R_a$, thus $M \models_w \prec a \succ L_0$, that is, $M \models_w \varphi_0 \to \prec a \succ L_0$; otherwise $M \models_w \varphi_1$, then $w \models_{PL} \varphi_1$. According to the construction of Σ^*, $w \models_{PL} \varphi_0$. Let $H_1 = \{L : \exists\, \varphi \to [a]L \in \Sigma(w \models_{PL} \varphi)\} \cup \{L_0\}$. We now prove that H_1 is consistent. To this end, suppose that there is a conflict f and $\neg f$ in H_1.

Case 1: $f = L_0$ or $\neg f = L_0$, say the former, there must exist a law $\varphi_2 \to [a]\neg f \in \Sigma$. According to the assumption of safety, $\vdash \neg\varphi_0 \vee \neg\varphi_1 \vee \neg\varphi_2$. We know that $w \models_{PL} \varphi_0 \wedge \varphi_1$, so $w \not\models_{PL} \varphi_2$. Thus $\neg f \notin H_1$, a contradiction.

Case 2: $f \neq L_0$ and $\neg f \neq L_0$, then there must be another law $\varphi_3 \to [a]f \in \Sigma$. According to the assumption of safety again, $\vdash \neg\varphi_1 \vee \neg\varphi_2 \vee \neg\varphi_3$. We know that $w \models_{PL} \varphi_1$, thus, $w \models \neg\varphi_2 \vee \neg\varphi_3$. This contradicts both f and $\neg f$ in H_1.

Secondly, we extend H_1 into H_2 such that for any fluent $f \in \mathbf{Flu_d}$, $f \in H_2$ iff $\neg f \notin H_2$. That means H_2 is an interpretation of $\mathbf{Flu_d}$. Next, let $H_3 = H_2 \cup \{L : \exists[\varphi]L \in \Sigma(H_2 \models_{PL} \varphi)\}$. It is not hard to prove that H_3 is consistent by assumption 2 of safety. Finally, we extend H_3 into an interpretation t of Flu. It is easy to see that $w' \in W$ and $(w, w') \in R_a$. Therefore, $M \models_w \varphi_0 \to \prec a \succ L_0$.

4. For any $\varphi_0 \to \langle a \rangle \top \in \Sigma$, suppose that $M \models_w \varphi_0$, that is, $w \models_{PL} \varphi_0$. According to the construction of Σ^*, for each $\varphi_1 \to \prec a \succ L \in \Sigma$, $w \models_{PL} \varphi_1$. By the proof of last step, there is $w' \in W$ such that $(w, w') \in R_a$, therefore $M \models_w \langle a \rangle \top$, or $M \models_w \varphi_0 \to \langle a \rangle \top$.

We conclude that M is a Σ-model. □

Proof of Theorem 3: Assume that Γ is not Σ-consistent, that is, $\Gamma \vdash^{\Sigma} \bot$. According to the proof of the theorem 2, there exists a Σ-model $M = (W, \mathcal{R}, V)$ such that

$$W = \{w : w \text{ is an interpretation of } Flu \text{ and for each } [\varphi]L \in \Sigma,\ w \models_{PL} \varphi \to L\}$$

Since $\Gamma \cup D(\Sigma)$ is consistent, there exists $w_0 \in W$ such that $M \models_{w_0} \Gamma$, a contradiction. Therefore Γ is Σ-consistent. □

Proof of Proposition 2: First we prove that for any interpretation I of $\mathbf{Flu}_d$, $I \cup D(\Sigma)$ is consistent. Let I' be an interpretation of $\mathbf{Flu}$ which is an extension of I and assigns to each fluent in $\mathbf{Flu}_i$ true. According to Assumption 2, $D(\Sigma)$ is true under I', so is $I \cup D(\Sigma)$. Thus $I \cup D(\Sigma)$ is consistent. Then by Theorem 3, each interpretation I of $\mathbf{Flu}_d$ is Σ-consistent. Therefore Σ is regionally consistent over $\mathbf{Flu}_d$. □

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