

# Game Theory in AI and MAS

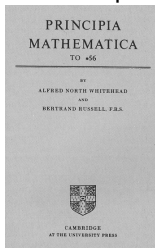
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30 August 2010  
Daegu, Korea

# AI Challenges

- Challenge 1: Build a machine that proves mathematical theorems.



- Challenge 2: Build a machine that plays chess.



picture from <http://www.thetech.org/>

# AI Challenges

- Challenge 3: Build a team of robots that plays soccer in the World Cup.



picture from <http://www.prweb.com/>

*“By mid-21st century, a team of fully autonomous humanoid robot soccer players shall win the game, complying with the official rule of the FIFA, against the winner of the most recent World Cup”.*

# Multi-agent systems (MAS)

A **multi-agent system (MAS)** is a system composed of multiple interacting intelligent agents, each of which is:

- Autonomous: *acts on the environment over time in pursuit of its own agenda.*
- Self-interested: *directs its activity towards achieving its goals.*
- Decentralized: *no designated controlling agent.*

*“The study of multi-agent systems (MAS) focuses on systems in which many intelligent agents **interact** with each other”.*

by Katia Sycara

# Game theory

“**Game theory** is a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers **interact**.”

by Osborne and Rubinstein



picture from <http://www.gsom.pu.ru/en/gtm2008/>

# Objectives of the tutorials

The aim of this tutorial is to briefly introduce the fundamental concepts of game theory and their applications in the areas of Artificial Intelligence and Multi-Agent Systems. We will focus on only those key models of game theory that have generated profound impact on the researches in AI and MAS, including equilibrium concepts, bargaining solutions, coalitional games and computational issues.

# Tutorial overview

- A model of multi-agent system
- Equilibrium analysis
- Coalitional games
- Bargaining solutions
- Computational issues
- Recommended readings

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# A model of multi-agent systems

Consider a **multi-agent system**  $(N, \{A_i\}_{i \in N}, S, \{\succeq_i\}_{i \in N}, \tau)$ , where

- $N$  is a set of **agents**, mostly represented as  $\{1, 2, \dots, n\}$ .
- $A_i$  is a non-empty set of **actions** available to agent  $i$ .
- $S$  is a set of **states** (possible outcomes of the system).
- $\succeq_i$  is a **preference relation** on  $S$ .
- $\tau : \prod_{i \in N} A_i \rightarrow S$  is a **state transition function** (a simplified version).

# Agent interaction

- An **encounter**, also called a **strategy profile**, is a joint action in which each agent chooses an action to perform. An encounter can be represented by  $\mathbf{a} = (a_1, \dots, a_n)$ , where  $a_i \in A_i$  is the action taken by agent  $i$ .
- State transition:

$$\tau : (a_1, \dots, a_n) \rightarrow s$$

which specifies the state change with each encounter.

- The **interaction** between agents can be modeled as a sequence of encounters.
- Given an initial state  $s_0$  and an encounter sequence  $\mathbf{a}^1, \mathbf{a}^2, \dots$ , we can observe the effect of interaction by the state transitions:

$$s_0 \xrightarrow{\mathbf{a}^1} s_1 \xrightarrow{\mathbf{a}^2} s_2 \xrightarrow{\mathbf{a}^3} s_3 \dots$$

where  $s_k = \tau(\mathbf{a}^k)$  ( $k = 1, 2, \dots$ ).

- In a multi-agent system, decision-making for an agent is much more complicated than in a single agent system because none of the agents in the multi-agent system can determine the outcome of the system. The system states depend on the joint actions of the agents.
- The fundamental assumptions that underlie the research of MAS are:
  - Each agent is self-interested: *directs its activity towards achieving its own goals.*
  - Each agent reasons strategically: *takes into account their knowledge or expectations of other decision-makers' behavior.*

- Preference over states:

$$s \succsim_i s', \text{ where } s, s' \in S.$$

- Preference over strategy profiles:

$$\mathbf{a} \succsim_i \mathbf{a}', \text{ where } \mathbf{a}, \mathbf{a}' \in \prod_{i \in N} A_i.$$

- An example: *Battle of the Sexes*

“A loved couple eat out every weekend. There are only two restaurants in the little town they live: one offers Chinese food and the other serves Korean food. They want to go together but the husband prefers Chinese food and the wife prefers Korean food.”

# Agent's preferences

- $N = \{h, w\}$ , where  $h$  represents the husband and  $w$  the wife.
- $A_i = \{\text{Chinese}, \text{Korean}\}$  for each  $i \in N$ .
- $S = \{s_0, (\text{Chinese}, \text{Chinese}), (\text{Korean}, \text{Korean}), (\text{Chinese}, \text{Korean}), (\text{Korean}, \text{Chinese})\}$ .
- The husband's preferences are:

$$\begin{aligned} &(\text{Chinese}, \text{Chinese}) \succ_1 (\text{Korean}, \text{Korean}) \succ_1 \\ &(\text{Chinese}, \text{Korean}) \sim_1 (\text{Korean}, \text{Chinese}). \end{aligned}$$

The wife's preferences are:

$$\begin{aligned} &(\text{Korean}, \text{Korean}) \succ_2 (\text{Chinese}, \text{Chinese}) \succ_2 \\ &(\text{Chinese}, \text{Korean}) \sim_2 (\text{Korean}, \text{Chinese}). \end{aligned}$$

# Representing agents' preferences

- Utility over states: **Utility function** (or **payoff function**) for agent  $i$ :

$$u_i : S \rightarrow \mathbb{R}$$

in the sense that

$$u_i(s) \geq u_i(s') \text{ if and only if } s \succsim_i s'$$

- Utility over strategic profiles:  
Utility function can also be defined on the set of encounters:

$$u_i : \prod_{i \in N} A_i \rightarrow \mathbb{R}$$

which gives the utility of agent  $i$  with respect to each strategy profile.

# Strategic reasoning

		agent 2	
		$a$	$b$
agent 1	$a$	$(4, 4)$	$(4, 1)$
	$b$	$(1, 4)$	$(1, 1)$

- Agent 1 will take action  $a$  because no matter what agent 2 does, it can always receive 4.
- Agent 2 will do the same for the same reason.
- Therefore  $(a, a)$  will be the outcome, which is called a **dominant strategy equilibrium**.

# Battle of the Sexes

In Battle-of-the-Sexes example, assume that the utilities of each agent are as follows:

$$u_w(\text{Korean}, \text{Korean}) = u_h(\text{Chinese}, \text{Chinese}) = 2$$

$$u_w(\text{Chinese}, \text{Chinese}) = u_h(\text{Korean}, \text{Korean}) = 1$$

$$u_i(\text{Chinese}, \text{Korean}) = u_i(\text{Korean}, \text{Chinese}) = 0 \text{ for each } i \in \{w, h\}.$$

		wife	
		Chinese	Korean
husband	Chinese	(2, 1)	(0, 0)
	Korean	(0, 0)	(1, 2)



# Strategic reasoning

For the Battle of the Sexes example, the payoff of one agent depends on the strategy of the other. No strategy is clearly better than the other for each agent.

		wife	
		Chinese	Korean
husband	Chinese	(2, 1)	(0, 0)
	Korean	(0, 0)	(1, 2)

Which restaurant the couple will go?

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# Nash Equilibrium

- An encounter (strategy profile)  $\mathbf{a}^* = (a_1^*, a_2^*, \dots, a_n^*)$  is a **Nash equilibrium** if for each agent  $i \in N$

$$(a_i^*, a_{-i}^*) \succsim_i (a_i, a_{-i}^*) \text{ for all } a_i \in A_i$$

or

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \text{ for all } a_i \in A_i$$

- It means that **no agent can profitably deviate, given the actions of the other agents.**

# Nash Equilibrium

- Battle of the Sexes:

		wife	
		Chinese	Korean
husband	Chinese	(2, 1)	(0, 0)
	Korean	(0, 0)	(1, 2)

The strategy profiles (*Chinese, Chinese*) and (*Korean, Korean*) are Nash Equilibria.

- A Nash equilibrium represents a steady state of a multi-agent system in which each agent holds the correct expectation about the other agents' behavior and acts rationally.

# Competitive and Cooperative Interactions

- The Prisoner's Dilemma

*"Two suspects in a crime are put into separate cells. If they both confess, each will be sentenced to three years in prison. If only one of them confesses, he will be freed and used as a witness against the other, who will receive a sentence of five years. If neither confesses, they will both be convicted of a minor offense and spend one year in prison."*

		agent 2	
		C	N
agent 1	C	(2, 2)	(5, 0)
	N	(0, 5)	(4, 4)

The Nash equilibrium is  $(C, C)$  rather than  $(N, N)$ .

# Competitive and Cooperative Interactions

- The concept of Nash equilibrium is actually based on the assumption that the agents are strictly competitive. A Nash equilibrium is not necessarily the “best” strategy for each agent but the stable ones for them.
- If cooperation among the agents exists, the outcome can be different.

# Competitive and Cooperative game theory

- Non-cooperative game theory:
  - Rules are complete.
  - The ultimate decision units are the individual players.
  - Commitments are not available, unless allowed for by the rules of the game.
  - The major tool is equilibrium analysis.
- Cooperative game theory:
  - Rules are kept implicitly.
  - The emphasis is on coalitions.
  - Commitments are available.
  - The major tool is axiomatic model.

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- A **coalitional game** is a pair  $G = \langle N, v \rangle$ , where
  - $N$  is a nonempty set of agents, each subset of  $N$  called a **coalition**, and  $N$  called **grand coalition**.
  - $v$  is a function from  $2^N$  to  $\mathbb{R}$ , satisfying  $v(\emptyset) = 0$ .

For each coalition  $S$ ,  $v(S)$  is the total payoff that is available for division among the members of  $S$ . Here the payoff is assumed to be transferable.

# Coalitional Games

- The Prisoner's Dilemma

		agent 2	
		<i>C</i>	<i>N</i>
agent 1	<i>C</i>	(2, 2)	(5, 0)
	<i>N</i>	(0, 5)	(4, 4)

- Coalitional game:  $G = \langle \{agent\ 1, agent\ 2\}, v \rangle$ , where  $v$  is defined as follows:

- $v(\{agent\ 1, agent\ 2\}) = 8.$
- $v(\{agent\ 1\}) = v(\{agent\ 2\}) = 2.$

$v(S)$  is the most payoff that the coalition  $S$  can guarantee independently of the behavior of the coalition  $N \setminus S$ .

- Will they cooperate?

# Solution concept: the core

- A **payoff allocation** is any vector  $\mathbf{x} = (x_i)_{i \in N}$  in  $\Re^N$ , which means that player  $i$  receives  $x_i$  as the utility payoff.
- A payoff allocation  $\mathbf{x}$  is **feasible for a coalition  $S$**  if  $\sum_{i \in S} x_i \leq v(S)$ .

This is because  $v(S)$  is the all available payoff for coalition  $S$ .

- A payoff allocation  $(x_i)_{i \in N}$  is **feasible** if it is feasible for the grand coalition  $N$ .
- A coalition  $S$  can **improve on** an allocation  $\mathbf{x}$  iff  $v(S) > \sum_{i \in S} x_i$ .  
In this case, we can find a feasible allocation for  $S$  which is strictly better than  $\mathbf{x}$ .
- An allocation  $\mathbf{x}$  is in the **core** of a coalitional game  $\langle N, v \rangle$  if  $\mathbf{x}$  is feasible and no coalition can improve on  $\mathbf{x}$ , i.e.,

$$\sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S), \forall S \subseteq N$$

# Solution concept: the core

- $G = \langle \{agent\ 1, agent\ 2\}, v \rangle$ , where  $v$  defined as follows:
  - $v(\{agent\ 1, agent\ 2\}) = 8$ .
  - $v(\{agent\ 1\}) = v(\{agent\ 2\}) = 2$ .
- The allocations  $(2, 6), (3, 5), \dots$  are in the core. In fact, any allocation  $(x_1, x_2)$  such that  $x_1 \geq 2, x_2 \geq 2$  and  $x_1 + x_2 = 8$  is in the core.
- To receive the payoff in the core, all the agents have to cooperate.

# From coalition to bargaining

- Many other solution concepts have also been proposed, such as stable set, kernel, bargaining set etc.
- All these solution concepts specify whether a set of agents should form a coalition for a better payoff. None of them tells us, within a coalition, who can receive more or less.
- A **bargaining theory** deals with the problem of how the agents in a coalition share the outcome of cooperation.

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  - Traditional bargaining solutions
  - A logic-based bargaining solution
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# Bargaining and multi-agent systems

*“The Nash bargaining solution is one of the most fundamental models in modern economic theory.”*

Ariel Rubinstein, 2000

*“A two-person bargaining situation involves two individuals who have the opportunity to **collaborate** for mutual benefit in more than one way.”*

by John Nash

*“Under such a definition, nearly all human **interaction** can be seen as bargaining of one form or another.”*

by Ken Binmore et al.

*“The study of multiagent systems (MAS) focuses on systems in which many intelligent agents **interact** with each other”.*

by Katia Sycara

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# Bargaining problems

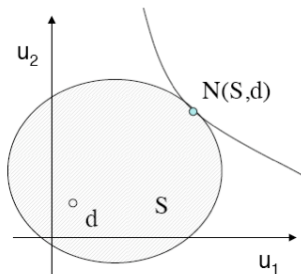
- **Bargaining problem:** Pie Devision.
  - Devision range:  $x \in [0, 1], y = 1 - x$
  - Utility of player 1:  $u_1(x)$
  - Utility of player 2:  $u_2(y)$



- **Bargaining game:**  $(S, d)$ , where  $S \subseteq \mathbb{R}^2$  &  $d \in S$
- **Bargaining solution:**  $f(S, d) \in S$

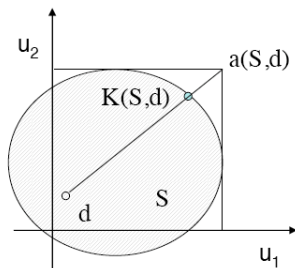
# Classical bargaining solutions

- Nash's solution (NS): the maximizer of the product of utilities.



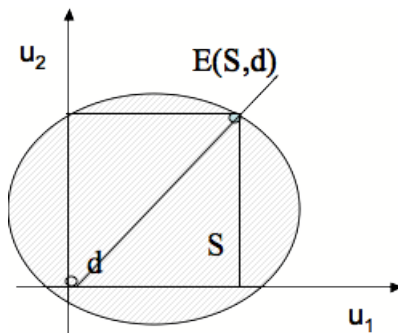
# Classical bargaining solutions

- **Kalai-Smorodinsky's Solution (KSS)**: the maximizer of the points in  $S$  on the segment connecting  $d$  and  $a(S,d)$ .



# Classical bargaining solutions

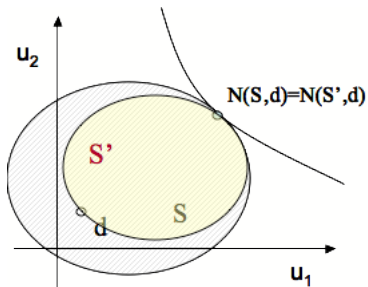
- **Egalitarian solution (ES)**: the maximal point with equal coordinates.



# Characterization of Bargaining Solutions

A bargaining solution is the NS iff it satisfies:

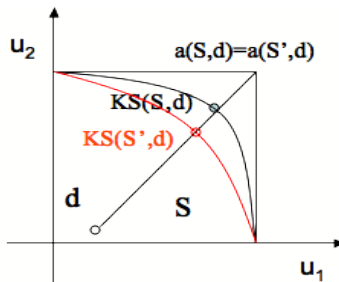
- Pareto-optimality
- Symmetry
- Scale invariance
- Independence of irrelevant alternatives.



# Characterization of Bargaining Solutions

A bargaining solution is the KSS iff it satisfies:

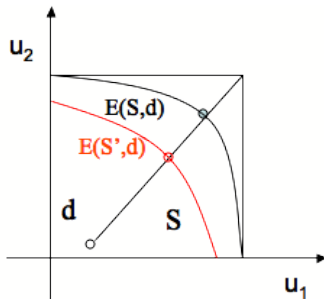
- Pareto-optimality
- Symmetry
- Scale invariance
- Restricted Monotonicity



# Characterization of Bargaining Solutions

A bargaining solution is the Egalitarian solution iff it satisfies

- Pareto Optimality,
- Symmetry and
- Strong monotonicity.



# An example: splitting a pie

## Example

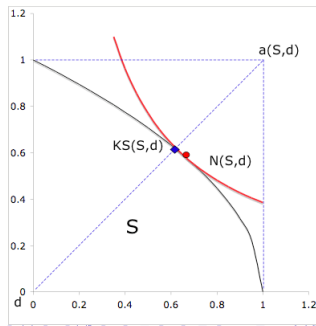
Two players bargain over the split of a pie.

$$u_1(x) = x$$

$$u_2(y) = \sqrt{y}$$

$$S = \{(x, \sqrt{y}) : 0 \leq x \leq 1 \text{ and } y = 1 - x\}, d = (0, 0).$$

- Nash's prediction: (66.7%, 33.3%)
- Kalai-Smorodinsky's prediction (same as Egalitarian): (61.8%, 38.2%)





# The “numbers” illusion

“Can this prediction be tested as in the sciences?”

“The use of numbers, even if analytically convenient, obscures the meaning of the model and creates the illusion that it can produce quantitative results.”

“I am not convinced that Nash’s theory has done more than clarify the logic of one consideration which influences bargaining outcomes. I can not see how this consideration will comprehensively explain real-life bargaining results.”

“Were game theorists to use a more natural language to specify the model, the solution would become clearer and more meaningful.”

[Ariel Rubinstein, 2000]

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## Example: Political negotiation

Two political parties in the Parliament bargain over a government rescue plan in response to the 2008 financial crisis. Proposals from the parties:

- Inject funds into struggling financial institutions
- Rescue car makers
- Relieve homeowners of heavy house mortgage
- Sponsor job training and job creation.
- Increase taxes

Obviously each party has their benefits from different industries therefore has preference on different rescue plans. However, representing the preference for each item in numbers can be a hard job for each party.

# Logical solution: a possibility

- Represent bargaining terms in logic:  
*bank*: fund financial institutions;  
*car*: rescue car makers;  
*house*: help house mortgagors;  
*training*: create training opportunities;  
*incTax*: increase taxes.
- Represent constraints in logic:  
 $\neg(\textit{bank} \wedge \textit{house})$ : mortgagees and mortgagors shouldn't be both funded.  
 $(\textit{car} \wedge \textit{bank}) \rightarrow \textit{incTax}$ : it is impossible to rescue both car industry and financial institutions without increasing taxes.

# Representation of bargaining problems: demands

## Bargainers' demands:

- Party A wants to inject almost all funds into the major banks but a small amount for job training. Tax increase is never a policy of party A.
- Party B insists on funding car makers and individual homeowners.
- Both parties know that there is no need to support both sides of house mortgage. Also the government budget does not allow to rescue both car industry and financial institutions unless increase taxes.

A:  $\{bank, training, \neg incTax, \neg(bank \wedge house), (car \wedge bank) \rightarrow incTax\}$

B:  $\{car, house, \neg(bank \wedge house), (car \wedge bank) \rightarrow incTax\}$

# Representation of bargaining problems: conflicts

A:  $\{bank, training, \neg incTax, \neg(bank \wedge house), (car \wedge bank) \rightarrow incTax\}$

B:  $\{car, house, \neg(bank \wedge house), (car \wedge bank) \rightarrow incTax\}$

Conflicts between the negotiation parties:

## Conflict 1

$\{bank, house, \neg(bank \wedge house)\} \vdash \perp$

## Conflict 2

$\{bank, car, \neg incTax, (car \wedge bank) \rightarrow incTax\} \vdash \perp$

# Representation of Preferences

Each party ranks their bargaining items in a total pre-order, representing the firmness the party insists on their demanded items (the higher the firmer).

## Party A

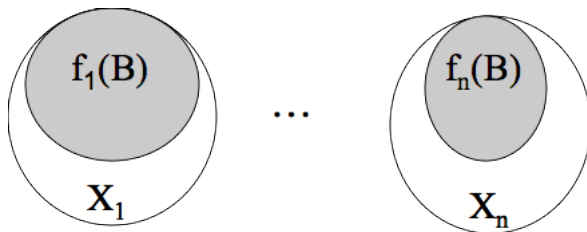
$\neg(\textit{bank} \wedge \textit{house})$
$(\textit{car} \wedge \textit{bank}) \rightarrow \textit{incTax}$
$\neg \textit{inTax}$
<i>bank</i>
<i>training</i>

## Party B

$\neg(\textit{bank} \wedge \textit{house})$
$(\textit{car} \wedge \textit{bank}) \rightarrow \textit{incTax}$
<i>car</i>
<i>house</i>

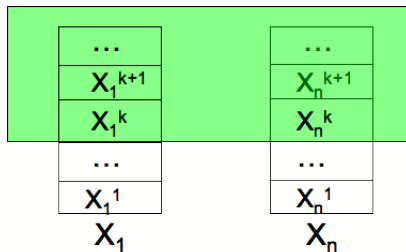
# Logical model of bargaining

- Bargaining game:  $G = ((X_1, \leq_1), \dots, (X_n, \leq_n))$ .
- Bargaining solution:  $f(G) = (C_1, \dots, C_n)$ , where  $C_i \subseteq X_i$ .
- Agreement:  $A(G) = \bigcup_{i \in N} f_i(B)$





# Solution construction: the simplest approach



$\neg(\text{bank} \wedge \text{house})$	$\neg(\text{bank} \wedge \text{house})$
$(\text{car} \wedge \text{bank}) \rightarrow \text{incTax}$	$(\text{car} \wedge \text{bank}) \rightarrow \text{incTax}$
$\neg \text{inTax}$	$(\text{car} \wedge \text{bank}) \rightarrow \text{incTax}$
<i>bank</i>	<i>car</i>
<i>training</i>	<i>house</i>

The solution is  $\{\neg(\text{bank} \wedge \text{house}), (\text{car} \wedge \text{bank}) \rightarrow \text{incTax}, \neg \text{inTax}\}$ , meaning that nothing is agreed.

# Solution construction: dummy demands

Party A	Party B
$\neg(bank \wedge house)$ $(car \wedge bank) \rightarrow incTax$	$\neg(bank \wedge house)$ $(car \wedge bank) \rightarrow incTax$
$\neg incTax$	$car$
$bank$	$house$
$training$	$coffee$

The solution is  $\{\neg(bank \wedge house), (car \wedge bank) \rightarrow incTax, \neg incTax, car\}$ , meaning that rescuing car makers has been agreed.

## Key idea

We can use dummy demands to represent intentional delays, which plays a similar role as non-linearity of utility functions.

# Characterization of the solution

The logic solution is characterized by the following axioms:

① **Consistency:**

$\bigcup_{i \in N} f_i(G)$  is consistent.

② **Comprehensiveness:**

For each  $i$ ,  $f_i(G)$  is comprehensive.

③ **Collective Rationality:**

If  $G$  is non-conflictive, then  $f_i(G) = X_i$  for all  $i$ .

④ **Disagreement:**

If  $G$  represents a disagreement situation, then  $f_i(G) = \emptyset$  for all  $i$ .

⑤ **Contraction independence:**

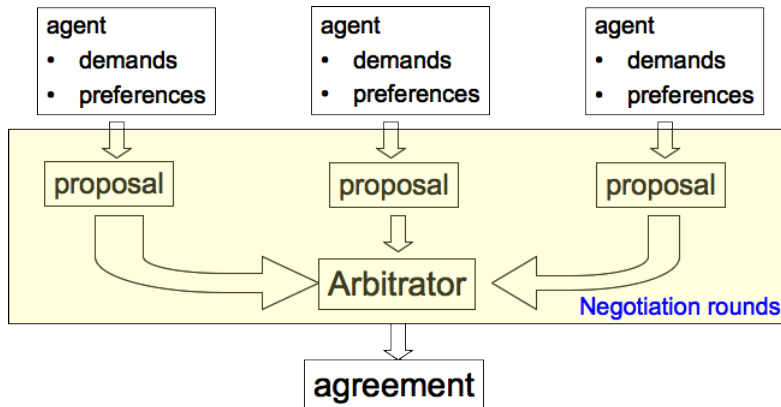
If  $G' \sqsubset_{max} G$ , then  $f(G) = f(G')$  unless  $G$  is non-conflictive.

# Comments on the logic-based solution

- A logical theory for multi-issue, n-person, raw domain bargaining
- Bargaining reasoning:
  - Identify conflicts.
  - Modeling risk.
- Representation of bargaining problems

Logical language + ordering  $>$  utility function
- The use of logical language leads to the derivation of a solution that resembles qualitative information and quantitative information.
- Related work [Zhang *et al.* 2004] [Zhang 2005] [Zhang and Zhang 2006] [Zhang 2007] [Zhang and Zhang 2008] [Zhang 2010].

# Automated negotiation



The arbitrator is not allowed to release any information of an agent to other agents unless he is authorized to do so.

# Tutorial overview

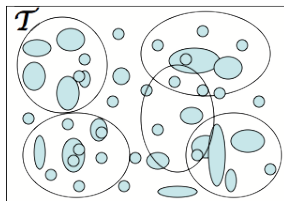
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# Apply bargaining solutions to AI: a case study

## Definition: [Zlotkin and Rosenschein 1993]

A *task oriented domain* (TOD) is a tuple  $\langle \mathcal{T}, \mathcal{A}, c \rangle$  where:

- 1  $\mathcal{T}$ : a set of possible tasks;
- 2  $\mathcal{A}$ : a set of agents;
- 3 Cost function  $c$ : define the cost of each set of tasks.



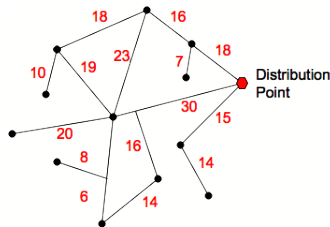
# Examples of TOD

## Example

**Delivery Domain:** A set of agents have to deliver sets of parcels to a number of cities, which are distributed on a weighted graph. They can exchange tasks at no extra cost while they are at the distribution point, prior to delivery.

Other examples [Zlotkin and Rosenschein 1994]:

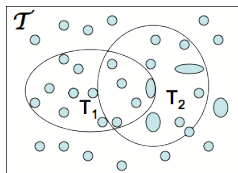
- Postmen domain
- Database queries
- Other task sharing problems with self-interested agents ...





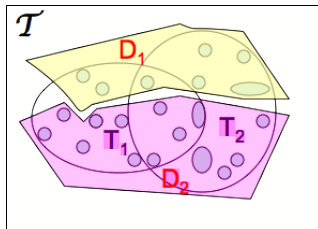
# Task oriented negotiation

- Consider a two-agent TOD ( $\mathcal{T}, \mathcal{A}, c$ ).
- An *encounter* within the TOD is a pair  $(T_1, T_2)$  such that  $T_1 \subseteq \mathcal{T}$  and  $T_2 \subseteq \mathcal{T}$ , representing the initial allocation of tasks to each agent.
- Problem: *How to redistribute the tasks to reduce the cost of execution based on the assumption that each agent is self-interested?*



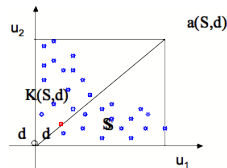
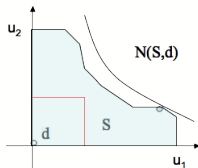
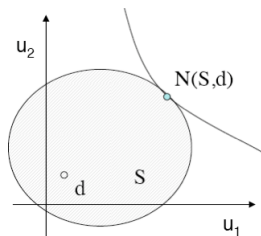
# Pure deals and utilities

- A *pure deal* of an encounter  $(T_1, T_2)$  is a pair  $D = (D_1, D_2)$  such that  $D_1 \cup D_2 = T_1 \cup T_2$ .
- Utility of a pure deal (cost reduction):  
 $u_1(D) = c(T_1) - c(D_1)$   
 $u_2(D) = c(T_2) - c(D_2)$
- disagreement:  $T = (T_1, T_2)$ .



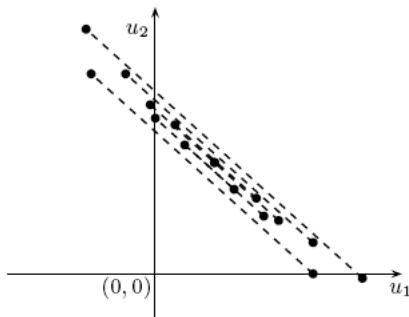
# Applicable domains

- Convex domain: guaranty of existence and uniqueness.
- Comprehensive: No guaranty of uniqueness.
- Finite domain: no guaranty of existence.
- Normal randomization is not applicable to TOD.



# Mixed deals

- $(D_1, D_2) : p$  is a *mixed deal* if  $p$  is a probability, meaning that agent 1 has  $p$  chance to do tasks  $D_1$  and  $1 - p$  chance to do tasks  $D_2$ .
- If  $(D_1, D_2) : p$  is a mixed deal,  $(D_2, D_1) : p$  is a mixed deal.
- The resulting domain is neither convex nor comprehensive.



# Bargaining solutions for TOD

- Nash solution:

$$f(T) = \arg \max_{D \in \bar{I}(T)} (C_1(T) - C_1(D))(C_2(T) - C_2(D))$$

- Egalitarian solution:

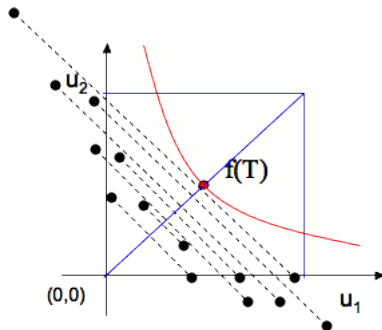
$$f(T) = \arg \max_{D \in \bar{I}(T)} \{v : C_1(T) - C_1(D) = C_2(T) - C_2(D) = v\}$$

- KS solution:

$$f(T) = \arg \max_{D \in \bar{I}(T)} \{v : C_1(T) - C_1(D) = a_1 v \text{ \& } C_2(T) - C_2(D) = a_2 v\}$$

## Theorem

*The above solutions give the same and unique outcome, modulo equivalence of costs.*



## Definition

Given a TOD, a *solution  $f$  in mixed deals* is a function that assigns to each encounter  $T$  a set of mixed deals.

Expected properties:

- No agent gains negative utility.
- The solution should not be empty.
- Deals with identical costs to each agent are treated the same.
- The solution should try to maximize the reduction of costs.
- The solution should try to minimize the imbalance of workload.

# Characterization of the solution

**Theorem:** Given a TOD, a negotiation function in mixed deals is the Egalitarian solution (therefore, the Nash solution and the KS solution) if and only if it satisfies the following axioms:

**IR:**  $D \in f(T)$  implies  $D \succeq T$ . (Individual rationality)

**NV:**  $f(T) \neq \emptyset$ . (Non-vacuity)

**Eq:**  $D \in f(T)$  and  $D \approx D'$  imply  $D' \in f(T)$ . (Equivalence)

**PO:** If  $D \in f(T)$ , there is no  $D'$  s.t.  $D' \succ D$ . (Pareto optimality)

**WB:** If  $D \in f(T)$ , there is no  $D'$  s.t.  $D' \triangleright_T D$ . (Workload balance)

$D' \triangleright_T D$  if and only if  $\text{dist}(D', T) < \text{dist}(D, T)$ , where

$$\text{dist}(D, T) = |(C_1(D) - C_2(D)) - (C_1(T) - C_2(T))|$$

## Definition

Given a TOD, a *solution  $f$  in pure deals* is a function that assigns to each encounter  $T$  a set of pure deals.

Expected properties:

- No agent gains negative utility.
- The solution should not be empty.
- Deals with identical costs to each agent are treated the same.
- The solution should try to maximize the reduction of costs.
- The solution should try to minimize the imbalance of workload.



# The pure deal solution and its characterization

**Theorem:** A pure deal solution satisfies the following axioms:

**IR:**  $D \in F(T)$  implies  $D \succeq T$ . (Individual rationality)

**NV:**  $F(T) \neq \emptyset$ . (Non-vacuity)

**Eq:**  $D \in f(T)$  and  $D \approx D'$  imply  $D' \in f(T)$ . (Equivalence)

**PO:** If  $D \in F(T)$ , there is no  $D'$  s.t.  $D' \succ D$ . (Pareto optimality)

**WB:**  $D \in F(T)$  and  $D' \triangleright_T D$  imply  $D' \notin P(T)$ . (Workload balance)

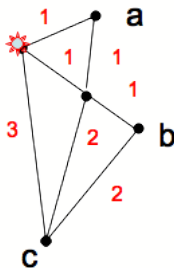
if and only if it is the function:

$$F(T) = \arg \min_{D \in NS(T)} dist(D, T),$$

where  $NS(T) = P(T) \cap I(T)$ .

# Example

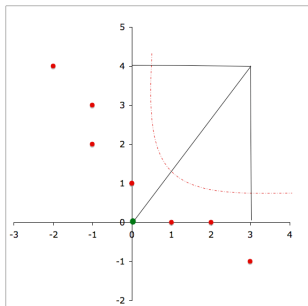
Consider a delivery problem in which two agents has parcels to be delivered to cities  $a$ ,  $b$  and  $c$ . Agent 1 has parcels to  $a$  and  $b$  while agent 2 has parcels to  $b$  and  $c$ , i.e.,  $T = \{\{a, b\}, \{b, c\}\}$ . The cost function is:  
 $c(\emptyset) = 0$ ,  $c(\{a\}) = 1$ ,  $c(\{b\}) = 2$ ,  $c(\{c\}) = 3$ ,  
 $c(\{a, b\}) = 3$ ,  $c(\{a, c\}) = 4$ ,  $c(\{b, c\}) = 4$ ,  $c(\{a, b, c\}) = 5$ .





# Solution in pure deals: with the classical definition

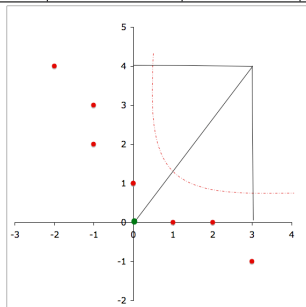
Pure deals	$(C_1, C_2)$	$(u_1, u_2)$
$(\emptyset, \{a, b, c\})$	$(0, 5)$	$(3, -1)$
$(\{a\}, \{b, c\})$	$(1, 4)$	$(2, 0)$
$(\{b\}, \{a, c\})$	$(2, 4)$	$(1, 0)$
$(\{c\}, \{a, b\})$	$(3, 3)$	$(0, 1)$
$(\{a, b\}, \{c\})$	$(3, 3)$	$(0, 1)$
$(\{b, c\}, \{a\})$	$(4, 1)$	$(-1, 3)$
$(\{a, c\}, \{b\})$	$(4, 2)$	$(-1, 2)$
$(\{a, b, c\}, \emptyset)$	$(5, 0)$	$(-2, 4)$



All individual rational pure deals, including the disagreement deal, are Nash solutions. The disagreement deal is the unique KS and Egalitarian solutions.

# Solution in pure deals: with the new definition

Pure deals	$(C_1, C_2)$	$(u_1, u_2)$	$dist(D, T)$
$(\{a\}, \{b, c\})$	$(1, 4)$	$(2, 0)$	2
$(\{b\}, \{a, c\})$	$(2, 4)$	$(1, 0)$	1
$(\{c\}, \{a, b\})$	$(3, 3)$	$(0, 1)$	1
$(\{a, b\}, \{c\})$	$(3, 3)$	$(0, 1)$	1



The pure deal solution is  $\{(\{c\}, \{a, b\}), (\{a, b\}, \{c\})\}$ .

# Comments on bargaining solutions

- Game-theoretic research on bargaining mostly motivated by economic problems.
- Bargaining problems from AI or MAS might require a different bargaining theory.
- Why axiomatic characterization of a bargaining solution is important?
- Reading materials: [Binmore *et al.* 1992] [Osborne and Rubinstein 1990] [Rosenschein and Zlotkin 1994] [Thomson 1994] [Zhang 2009].

# Tutorial overview

- A model of multi-agent systems
- Equilibrium analysis
- Bargaining solutions
- Coalitional games
- Computational issues
- Recommended readings

# Computational issues of game theory

- Game theory is to develop solution concepts for strategic interactions among intelligent agents. If a solution concept is not efficiently computable, it becomes less useful to the development of multi-agent systems.
- Game theory has been a fundamental research theme in economics and mathematics for several decades. However, the computational properties of game-theoretical problems had not been well-studied until the involvement of computer scientists.
- It has been discovered that many traditional game-theoretical solutions are computationally challenging, which motivates researches on computation-friendly alternative solutions.



# Examples of computationally intractable problems

- Finding a mixed Nash equilibrium is PPAD-complete (PPAD for polynomial parity argument, directed version).
- The mechanism design problem for deterministic mechanisms is NP-complete.
- The Winner Determination Problem for combinatorial auctions is NP-complete.
- Dynamic auction for combinatorial auctions is NP-complete.

# Handling intractability

- Find alternative solutions.
- Find approximations.
- Find tractable subclasses.

# Dynamic auction: a tractable procedure

- A case study of handling intractability.
- Scenario: A seller wishes to sell a set of indivisible items to a number of buyers. Each buyer has a private value over each bundle of items.
- The model:  $E = (N \cup \{0\}, X, \{v_i\}_{i \in N})$ , where
  - $N = \{1, 2, \dots, n\}$  is the set of buyers
  - 0 represents the seller
  - $X$  is the set of items
  - $v_i : 2^X \rightarrow \mathbb{Z}^+$  the buyer  $i$ 's value function
- Example:  $N = \{1, 2\}$ ,  $X = \{a, b\}$ .  
 $u_1(\emptyset) = 0$ ,  $v_1(\{a\}) = v_1(\{b\}) = v_1(\{a, b\}) = 1$ .  
 $u_2(\emptyset) = 0$ ,  $v_2(\{a\}) = v_2(\{b\}) = 1$ ,  $v_2(\{a, b\}) = 3$ .
- The problem: How to allocate the items to the buyers so that each item goes to the buyer who gives it the highest value?

- Allocation:  $\pi : N \cup \{0\} \rightarrow 2^X$ , which allocate a bundle of items to each buyer. One item can only be allocated to at most one buyer.
- Efficient allocation  $\pi^*$ :  $\pi^*(0) = \emptyset$  and for every allocation  $\pi$  of  $X$ ,

$$\sum_{i \in N} v_i^*(\pi(i)) \geq \sum_{i \in N} v_i(\pi(i))$$

- The problem: **How to find an efficient allocation?**

# Walrasian equilibria

- Price vector  $\mathbf{p}$ : assign a non-negative real number to each item in  $X$ .
- Demand correspondence:  $D_i(\mathbf{p}) = \arg \max_{A \subseteq X} (V_i(A) - \sum_{a \in A} p_a)$ ,  
representing all the bundles that give  $i$  the highest utility based on the current market price.

For instance, if  $\mathbf{p} = (0.5, 0.5)$ ,

$$D_1(\mathbf{p}) = \{\{a\}, \{b\}\}. \quad D_2(\mathbf{p}) = \{\{a, b\}\}$$

- Walrasian equilibrium  $(\mathbf{p}, \pi)$ :  $\mathbf{p}$  is a price vector and  $\pi$  is an allocation of  $X$  such that  $\pi(0) = \emptyset$  and  $\pi(i) \in D_i(\mathbf{p})$  for all  $i \in N$ .
- Any Walrasian equilibrium determines an efficient allocation.
- The problem: **How to find a Walrasian equilibrium?**

A dynamic auction procedure:

- ① Initially set the price vector  $\mathbf{p}$  to a starting price vector  $\mathbf{p}^0$ .
- ② Ask each buyer  $i$  to report her demand correspondence  $D_i(\mathbf{p})$ .
- ③ The seller makes a decision to the following problems:
  - Efficient allocation: determine if an efficient allocation exists. If yes, stop.
  - Price adjustment: determine which items have excess demand (positive or negative). Reset the prices of the items and go back to step (2).

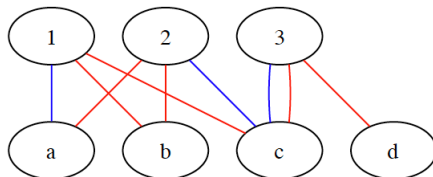
# Some results

## Proposition (bad news)

The efficient allocation problem is NP-complete.

Let  $N = \{1, 2, 3\}$  and  $X = \{a, b, c, d\}$ .

$D_1 = \{\{a\}, \{b, c\}\}$ ,  $D_2 = \{\{a, b\}, \{c\}\}$ ,  $D_3 = \{\{c\}, \{c, d\}\}$



## Proposition [Gul and Stacchetti 1999] (“good news”)

Walrasian equilibria do not always exist.

# Gross substitutes and complements condition

- Gross substitutes (GS) condition: if the prices of items were increased, the buyer would still want to buy the items the prices of which have not increased [Kelso and Crawford 1982].
- If each buyers valuation function satisfies GS, a Walrasian equilibrium exists, which can be found by a dynamic auction procedure [Gul and Stacchetti 2000].
- Gross substitutes and complements (GSC) condition: if all the selling items can be divided into two categories, say software and hardware, increasing the prices of items in one category and decreasing the prices of items in the other category would not affect the demand of the items which prices are not changed [Sun and Yang 2006].
- Any economy that satisfies GSC has a Walrasian equilibrium [Sun and Yang 2009].



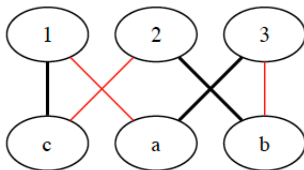
# Gross substitutes and complements condition

- GSC is the most general condition that guarantees the existence of Walrasian equilibria.
  - GS introduced by Kelso and Crawford in Econometrica 1982.
  - GSC introduced by Sun and Yang in Econometrica 2006.
- The problem: If an economy satisfies GSC, whether there is a polynomial algorithm to find a Walrasian equilibrium?

# Efficient allocation and bipartite matching

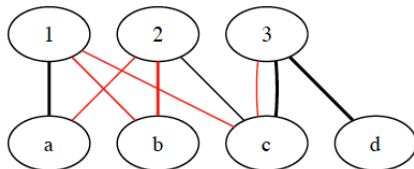
- Bipartite matching: unit demand

$$D_1 = \{\{a\}, \{c\}\}, D_2 = \{\{b\}, \{c\}\}, D_3 = \{\{a\}, \{b\}\}$$



- Bipartite quasi-matching: partial satisfaction.

$$D_1 = \{\{a\}, \{b, c\}\}, D_2 = \{\{a, b\}, \{c\}\}, D_3 = \{\{c\}, \{a, d\}\}$$



# Maximum quasi-matching algorithm

- If a valuation function  $v$  satisfies GSC, the demand correspondence  $D(\mathbf{p})$  will be the base of a **matroid** on  $X$ .
- If  $D(\mathbf{p})$  is the base of a matroid, then you can switch from one bundle to another bundle by just swapping one element.
- Use the augmentation technique to find a maximum quasi-matching.
- A maximum quasi-matching can be found in polynomial time.

# Double-direction auction algorithm

- 1 Announce an initial price vector  $\mathbf{p}^0$ .
- 2 At round  $t$ , ask each buyer  $i$  to submit her demand correspondence  $D_i(\mathbf{p}^t)$ .
- 3 Calculate a maximum quasi-matching  $M$ .
- 4 If  $M$  determines an efficient allocation, stop; Otherwise, adjust the price of each item so that the price of each under-demanded item is decreased by 1 and the price of each over-demanded item is increased by 1.
- 5 Go to step 2 for next round.

## Theorem

If an economy satisfies GSC and the initial price is set to 0 for one category and maximum for the other category, the algorithm converges to a Walrasian equilibrium.

# Summary of the case study

- The problem of finding a Walrasian equilibrium in general is NP-hard.
- We only have to deal with the markets where Walrasian equilibria exist.
- If the economy satisfies GSC, the problem can be solved in polynomial time.
- The overall complexity of finding a Walrasian equilibrium with Double-Direction Auction procedure is in  $O(|N \cup D \cup X|^4)$ .
- Related reading materials: [Sun and Yang 2009] [Zhang *et al.* 2010]

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# Recommended readings

- [Binmore *et al.* 1992] Ken Binmore, Martin J. Osborne, and Ariel Rubinstein. Noncooperative models of bargaining. In R.J. Aumann and S. Hart, editors, *Handbook of Game Theory with Economic Applications*, volume 1, chapter 7, pages 180-225. Elsevier, 1992.
- [Gul and Stacchetti 1999] Faruk Gul and Ennio Stacchetti. Walrasian equilibrium with gross substitutes. *Journal of Economic Theory*, 87(1):95-124, 1999.
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## Recommended readings

- [Zhang and Zhang 2008] Dongmo Zhang and Yan Zhang. An ordinal bargaining solution with fixed-point property. *Journal of Artificial Intelligence Research*, 33:433 - 464, 2008.
- [Zhang 2009] Dongmo Zhang. Axiomatic characterization of task oriented negotiation. In *Proceedings of the 21st International Joint Conference on Artificial Intelligence (IJCAI-09)*, 367372, 2009.
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- [Zhang et al. 2010] Dongmo Zhang, Laurent Perrussel, Wei Huang, Dynamic auction: a tractable auction procedure, in *Proceedings of the 24th AAAI Conference on Artificial Intelligence (AAAI-10)*, 935-940, 2010.

# The End

Thank you for your attention!