A Sequential Model for Reasoning about Bargaining in Logic Programs*

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Abstract. This paper presents a sequential model of bargaining based on abductive reasoning in ASP. We assume that each agent is represented by a logic program that encodes the background knowledge of the agent. Each agent has a set of goals to achieve but these goals are normally unachievable without an agreement from the other agent. We design an alternating-offers procedure that shows how an agreement between two agents can be reached through a reasoning process based on answer set programming and abduction. We prove that the procedure converges to a Nash equilibrium if each player makes rational offer/counter-offer at each round.

Keywords: bargaining theory, logic programming, sequential model.

1 Introduction

Bargaining has been a central research theme in economics for many decades and recently becomes an attractive research topic in artificial intelligence mainly driven by the advance of e-commence and multi-agent systems [1,2]. Different from other disciplines, the research on bargaining in artificial intelligence focuses more on reasoning mechanisms of bargaining process. A number of logical frameworks have been proposed in the literature for modelling different aspects of bargaining reasoning [3,4,5].

There are two different models of bargaining - *cooperative* and *non-cooperative*- that have been proposed in game theory. The cooperative model represents a bargain problem as a one-shot game and specifies the properties of bargaining solutions in an axiomatical system [1]. The noncooperative model of bargaining models a bargaining process as a sequential procedure. To specify bargaining reasoning, both models have been reformulated in logical frameworks. Zhang in [5] has proposed an axiomatic model of bargaining based on propositional logic. Several other authors have also constructed a range of different logic-based sequential models specifying bargaining reasoning, based on either argumentation, propositional logic or logic programming [3,4,6]. However, each of these models has limitation in either reasoning power or game-theoretic properties. The models that describe a bargaining situation in propositional formulas normally treat a formula as a whole therefore either keep a whole formula or drop a formula

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(logic is used for consistency maintenance only) [5,6]. The models based on argumentation or logic programs allow break of a formula for bargaining reasoning but the procedures that have been proposed normally lack of game-theoretic properties, such as convergency and pareto optimality [3,4]. This paper proposes a new sequential model of bargaining that specifies procedures of bargaining reasoning in answer set programming. We assume that each agent is represented by a logic program that encodes the background knowledge the agent uses for its bargaining reasoning. Each agent has a set of goals to achieve but these goals are normally unachievable without an agreement from the other agents. We design an alternating-offers procedure that shows how an agreement between two agents can be reached through a reasoning process based on answer set programming and abduction. We prove that the procedure converges to a Nash equilibrium if each player makes rational offer/counter-offer at each round.

The rest of the paper is organised as follows. Section 2 introduces our bargaining model. Section 3 presents the framework of our sequential bargaining model. Section 4 provide a construction of the sequential bargaining procedure and proves its equilibrium properties. The final section concludes the paper.

2 The Bargaining Model

In this section, we introduce a bargaining model in which each agent is equipped with a logic program as its background knowledge for bargaining reasoning and a set of goals to achieve. To make the framework simple we only consider the bargaining situations in which there are only two players.

Assume that $\mathcal{L}$ is a propositional language with finite number of propositional symbols (atoms). A literal can be either a positive atom, say a, or a negative atom, say $\neg a$. a and $\neg a$ are called complementary literals. A set of literals S is *consistent* if it contains no complementary literals; otherwise it is *inconsistent*. A *rule* is a formula

$$L_0 \leftarrow L_1, ..., L_m, not\ L_{m+1}, ..., not\ L_n (0 \leq m \leq n), \tag{1}$$

where each $L_i (0 \leq i \leq n)$ is a literal, *not* is *negation as failure* . We denote its head, positive body and negative body by $Head(r) = \{L_0\}$, $Pos(r) = \{L_1, \ldots, L_m\}$ and $Neg(r) = \{L_{m+1}, \ldots, L_n\}$ respectively. r is called a *fact* if $Pos(r) = \emptyset$ and $Neg(r) = \emptyset$. r is a *constraint* if $Head(r) = \emptyset$.

An answer set program is a finite set of *rules*. For a given *logic program* Π, we write $Head(\Pi) = \bigcup_{r \in \Pi} Head(r)$, $Pos(\Pi) = \bigcup_{r \in \Pi} Pos(r)$, $A(\Pi) = Pos(\Pi) \setminus Head(\Pi)$ and $\Pi \cup X = \Pi \cup \{L \leftarrow |L \in X\}$ where X is a set of literals. We use $AS(\Pi)$ to denote the set of consistent answer sets of a *logic program* Π.

In a bargaining situation, an agent might have a number of goals to achieve through the bargaining process. The aim of the agent is to reach an agreement with the other agent so that the other agent agrees the conditions that achieve his goals or some of his goals. If an agent cannot achieve all his goals, the agent might have a preference over these goals. A *model of player* includes an agent's knowledge, bargaining goals and its preference among these goals. The following definition gives such a *model of players*.

Definition 1. *A* two-player bargaining game *is a tuple* $M = ((\Pi_1, G_1, \leq_1), (\Pi_2, G_2, \leq_2))$*, where, for each i,* Π_i *is a logic program,* G_i *is a set of goals, each goal consisting of a set of literals, and* $\leq_i$ *is a total order over* G_i*.*

As a convention, we refer the opponent of player i as $-i$ in the sequnt. Given a logic program, a goal is achieved by the logic program if it is in an answer set of the program. However, if it is not achieved, we may wonder what are the conditions that can make the goal true. We call a set of conditions that achieves a goal under a logic program a *support*. In setting of our bargaining model, the concept of supports is important because if a condition that cannot be satisfied by one agent could be satisfied by another agent; an agent may request another agent to satisfy a condition by offering a condition the other agent is needed.

Definition 2. *Given a logic program Π and a set X of literals, we say $\Delta \subseteq A(\Pi)$ is a* minimal support for achieving *X from Π if it satisfies:*

1. $X \cap \Delta = \emptyset$.
2. *There is an answer set $S \in AS(\Pi \cup \Delta)$ such that $X \subseteq S$.*
3. *There is no $\Delta' \subset \Delta$ such that Δ' also satisfies Condition (1) and (2).*

We use $\alpha(\Pi, X)$ to represent the set of all possible minimal supports with respect to Π and X.

Given a bargaining game M, an *offer* of an agent is a pair (D, P), where $D \subseteq Lit$ and $P \subseteq Lit$. The set of all the possible offers is denoted by $\mathcal{O}$. Intuitively, an offer of a player represents the player's demands from the bargaining and the conditions he promises to the other player. D represents the current demands of the player and P denotes the current promises of the player to the other player.

Definition 3. *Let $M = ((\Pi_1, G_1, \leq_1), (\Pi_2, G_2, \leq_2))$ be a bargaining model. An offer $O = (D, P)$* achieves *player i's goal $g \in G_i$ if*

1. *$D \in \alpha(\Pi_i, g \cup P)$;*
2. *There is no $g' \in G_i$ such that g' satisfies condition (1) and $g <_i g'$.*

For each player i, let $\mathcal{G}_i : \mathcal{O} \to G_i \cup \{\emptyset\}$ such that for any $O \in \mathcal{O}$, $\mathcal{G}_i(O) = g$ if O of player i *achieves* a goal $g \in G_i$; otherwise, $\mathcal{G}_i(O) = \emptyset$.

For convenience, we assume that for each player i, $\emptyset \notin G_i$, that is, a goal cannot be empty. In addition, we extend the ordering relation $\leq_i$ to $G_i \cup \{\emptyset\}$ such that $\emptyset <_i g$ for all $g \in G_i$.

Definition 4. *For each player i, define an order $\preceq_i$ over $\mathcal{O}$ as follows:*

$$O' \preceq_i O'' \text{ iff } \mathcal{G}_i(O') \leq_i \mathcal{G}_i(O'')$$

We say that O'' *dominates* O' if $O' \preceq_i O''$. Since $\leq_i$ is a total order over $G_i \cup \{\emptyset\}$, it is easy to see that $\preceq_i$ is a total preorder over $\mathcal{O}$.

A player not only has assess each offer he mades to see which goal he can achieve if the offer is accepted but also has to assess the opponent's offer to check if the offer should be accepted. The way of assessing opponent's offers is the following: A goal g of player i is *achievable* with an offer $O_{-i} = (D_{-i}, P_{-i})$ from the opponent of player i if there is a counter-offer $O = (D, P)$ to his opposite such that O achieves g meanwhile $P = D_{-i}$ and $P_{-i} \subseteq D$. We denote $\mathcal{I}_i(O)$ as the maximal goal of player i that is *achievable* with the offer O from player $-i$.

3 Sequential Bargaining Procedures

We design a sequential bargaining procedure as follows. Two players i and $-i$ take actions only at times in the set $T = \{1, 2, \cdots\}$. In each round $t \in T$, one of the players, say i, makes an offer (D^t, P^t) (a member of $\mathcal{O}$), where D^t contains all the items that player i wants the player $-i$ to accept and P^t contains all the items that player i accepts (initially is empty). Then the play passes to round $t+1$; in this round player $-i$ makes a counter-offer (D^{t+1}, P^{t+1}). A player can terminate the procedure any time either set $D^t = P^{t-1}$ and $P^t = D^{t-1}$, in which case an agreement is reached or say nothing, in which case the game ends with a disagreement. The game continues whenever a player makes a new offer and the play passes to the next round [1].

Following the standard game-theoretical definition of bargaining procedures [1], we define a sequential bargaining procedure as follows. The extensive game of a sequential bargaining is a tuple $(N, H, P, \preceq_i)$ where

1). $N = \{1, 2\}$ is the set of players.

2). H is the set of histories. Each $h \in H$ is a sequence of offers that satisfies the following properties:

2.1). The empty sequence $\emptyset$ is a member of H.

2.2). If $(O^k)_{k=1}^K \in H$ and $L < K$, then $(O^k)_{k=1}^L \in H$.

A history $(O^k)_{k=1}^K \in H$ is *terminal* if there is no O^{K+1} such that $(O^k)_{k=1}^{K+1} \in H$. The set of terminal histories is denoted Z.

3). P is a function that assigns to each nonterminal history a number of N such that $P(h) = 1$ if the length of h is an even number and $P(h) = 2$ if the length of h is an odd number.

4). $\preceq_i$ is a preference relation on Z such that for any two histories $h = (O^k)_{k=1}^K \in Z$ and $h' = (O'^k)_{k=1}^{K'} \in Z$, $h \preceq_i h'$ if and only if $O^K \preceq_i O'^{K'}$.

5). For any $t_1, t_2 \in T$, $O^{t_2} = (D^{t_2}, \emptyset)$ and $O^{t_1} = (D^{t_1}, \emptyset)$ are two offers of player $i (i = 1 \ or\ 2)$. If $\mathcal{G}_i(O^{t_2}) <_i \mathcal{G}_i(O^{t_1})$, then $t_1 < t_2$. If $t_1 < t_2$, then $\mathcal{G}_i(O^{t_2}) \leq_i \mathcal{G}_i(O^{t_1})$.

6). For any $O^k (k > 1)$, if $O^k = (D^k, \emptyset)$, then $O^{k-1} \neq O^{k+1}$.

Let $A(h) = \{O : (h, O) \in H\}$. We then can define strategies of a player.

Definition 5. *A strategy, s_i, of player $i \in N$ in the extensive game of sequential bargaining is a function that assigns an offer in $A(h)$ to each nonterminal history $h \in H \backslash Z$ for which $P(h) = i$. A pair $s = (s_1, s_2)$ of strategies is called a strategy profile if for each $i \in \{1, 2\}$, s_i is a strategy of player i.*

Definition 6. *A pair of strategies (s_1, s_2) is a Nash equilibrium if, given s_2, no strategy of player 1 results in an outcome that player 1 prefers to the outcome generated by (s_1, s_2), and, given s_1, no strategy of player 2 results in an outcome that player 2 prefers to the outcome generated by (s_1, s_2).*

Nash equilibrium is an important measurement to judge whether a bargaining procedure is designed reasonable or not. The following section will introduce a concrete bargaining procedure and prove that the procedure converges to a Nash equilibrium in finite steps.

4 Construction of Bargaining Procedure

We now give a concrete algorithm to model the bargaining procedure between two players i and $-i$ using abductive reasoning. For convenience, we say g is the best goal of G if $g \in G$ and for any $g' \in G$, $g' \leq g$, which is denoted $B(G)$. We use G_i^t to represent the set of goals of player i at the t round. Let $M = ((\Pi_1, G_1, \leq_1), (\Pi_2, G_2, \leq_2))$ be a bargaining model. Assume that player $-i$ puts forward the first offer $O^1_{-i} = (D^1_{-i}, P^1_{-i})$.

Algorithm 1. constructing bargaining procedure with abductive method

Input: $\Pi_i (i = 1, 2)$, $G_i (i = 1, 2)$
Output: O_1 and O_2

$t := 1$; $G^1_{-i} := G_{-i}$; $G^0_i := G_i$;
$\mathcal{H}_{-i} := Initialize(\Pi_{-i}, G^1_{-i})$; $O^1_{-i} := \mathcal{H}_{-i}.top()$; $\mathcal{H}_i := Initialize(\Pi_i, G^0_i)$;
repeat
    $t := t + 1$;
    $O^t_i := CounterOffer(O^{t-1}_{-i})$;
    $O_i := O^t_i$;
    if $D^t_i = P^{t-1}_{-i}$ *and* $D^{t-1}_{-i} = P^t_i$ **then**
        break;
    end
    if $P^{t-1}_{-i} = \emptyset$ *and* $P^t_i = \emptyset$ **then**
        $\mathcal{H}_{-i} := \mathcal{H}_{-i} \setminus \{\mathcal{H}_{-i}.top()\}$;
        if $\mathcal{H}_{-i} = \emptyset$ **then**
            $G^{t-1}_{-i} := G^{t-1}_{-i} \setminus \{B(G^{t-1}_{-i})\}$;
            $\mathcal{H}_{-i} := Initialize(\Pi_{-i}, G^{t-1}_{-i})$;
        end
    end
    swap i and $-i$;
until $G^t_i = \emptyset$;

Procedure $Initialize$

Input: Π, G
Output: $\mathcal{H}$

for $\Delta \in \alpha(\Pi, B(G))$ **do**
    $O := (\Delta, \emptyset)$;
    $\mathcal{H}.push(O, G)$;
end

Given $M = ((\Pi_1, G_1, \leq_1), (\Pi_2, G_2, \leq_2))$ be a sequential bargaining model. The sequential bargaining procedure satisfies the following properties:

Proposition 1. *1. (*Mutual commitment*) For any $t \in T$, $P^t_i \subseteq D^{t-1}_{-i}$ and $P^{t-1}_{-i} \subseteq D^t_i$.*

*2. (*Individual rationality*) For any $t \in T$, if $O^t_i = (D^t_i, P^t_i)$ is a counter-offer of $O^{t-1}_{-i} = (D^{t-1}_{-i}, P^{t-1}_{-i})$, then $(P^{t-1}_{-i}, D^{t-1}_{-i}) \preceq_i O^t_i$.*

*3. (*Satisfactorily*) For any $t \in T$, if $P^t_i = D^{t-1}_{-i}$ and $D^t_i = P^{t-1}_{-i}$, then $D^{t+1}_{-i} = D^{t-1}_{-i}$.*

*4. (*Honest*) (1)For any $t_1, t_2 \in T$, let $O^{t_2} = (D^{t_2}_i, \emptyset)$ and $O^{t_1} = (D^{t_1}_i, \emptyset)$. If $\mathcal{G}_i(O^{t_2}) <_i \mathcal{G}_i(O^{t_1})$, then $t_1 < t_2$. If $t_1 < t_2$, then $\mathcal{G}_i(O^{t_2}) \leq_i \mathcal{G}_i(O^{t_1})$. (2)For any $t \in T (t > 1)$, if $O^{t-1}_{-i} = (D^{t-1}_{-i}, \emptyset)$, then $O^t_i \neq O^{t-2}_i$.*

Procedure $CounterOffer$

Input: O_{-i}^{t-1}
Output: O_i^t
if $D_{-i}^{t-1} \neq P_i^{t-2}$ **then**
  if $B(G_i^{t-2}) \leq_i \mathcal{I}_i(O_{-i}^{t-1})$ **then**
    foreach $\Delta \in \alpha(\Pi_i, \mathcal{I}_i(O_{-i}^{t-1}) \cup D_{-i}^{t-1})$ *such that* $P_{-i}^{t-1} \subseteq \Delta$ **do**
      $G_i^t := G_i^{t-2} \cup \{\mathcal{I}_i(O_{-i}^{t-1})\}$;
      $O_i' := (\Delta, D_{-i}^{t-1})$;
      $\mathcal{H}_i.push(O_i', G_i^t)$;
    end
  end
else
  if $P_{-i}^{t-1} \neq D_i^{t-2}$ **then**
    $\mathcal{H}_i := \mathcal{H}_i \setminus \{\mathcal{H}_i.top()\}$;
    if $\mathcal{H}_i = \emptyset$ **then**
      $G_i^t := G_i^{t-2} \setminus \{B(G_i^{t-2})\}$;
      $\mathcal{H}_i := Initialize(\Pi_i, G_i^t)$;
    end
  end
end
$O_i^t := \mathcal{H}_i.top()$;

Theorem 1. *Given any bargaining model, Algorithm 1 generates a strategy profile in finite steps that is a Nash equilibrium under Definition 7.*

5 Conclusion

We have proposed a sequential model of bargaining based on abductive reasoning in ASP and devised a bargaining procedure to demonstrate how two agents reach an agreement through abductive reasoning. We have shown that the sequential bargaining procedure converges a Nash equilibrium. We have also shown that the procedure satisfies a number of desirable properties.

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